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On the manipulability of the constrained Gale–Shapley and Boston school choice mechanisms. Part 2. Comparing harm of manipulations¹

Abstract. This work presents a simulation-based comparative analysis on the harm of manipulations in two widely used school choice mechanisms – the constrained Boston and Gale–Shapley – through the perspective of the share (percentage) of students getting into schools and the average welfare of students. Thus, this part of the research extends the manipulability analysis presented in the first part but analyzes not the vulnerability of mechanisms to manipulations but the harm of manipulations per se. We also investigate the connections between the parameters of the problems (percentage of sophisticated students and the mechanism constraint, i.e. the maximum number of schools that students are allowed to list in their preferences) and the outcomes of the mechanisms. Finally, we analyze and compare two different mechanism designs: the one where students submit their preferences by the same point in time and the one where students are allowed to change their preferences through a certain common period of time. In this part of the research we show two main advantages of the Gale–Shapley mechanism compared to the Boston mechanism: the higher percentage of students getting into schools and, under the realistic assumption of correlated preferences of students, higher average welfare of sincere students, with the average welfare of all students being statistically equal under two mechanisms in the most of analyzed scenarios.

Keywords: *market design, school choice, manipulability.*

JEL Classification: C78, D47, D78.

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1. Introduction

In 2003, A. Abdulkadiroğlu and T. Sönmez, in their seminal paper, showed manipulability of the Boston mechanism, which was the most common mechanism in school choice practice at the time, and suggested two well-known mechanisms (in matching literature) as potential replacements for the Boston mechanism – the Gale–Shapley (GS) and the Top Trading Cycles (TTC) mechanisms. Since then, the manipulable Boston mechanism was replaced by the strategy-proof Gale–Shapley mechanism in many school choice systems. However, the Gale–Shapley mechanism is non-manipulable only if students are able to submit their full list of preferences. In practice this is usually not the case. And instead constrained versions of mechanisms are implemented – Chicago Public Schools (CPS) admission system², Denver Public Schools admission system³, admission systems in London and many others. In con-

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² <https://www.cps.edu/schools/apply-enroll/>

³ <https://www.dpsk12.org/>

strained mechanisms, students submit only their top k schools instead of submitting preferences over all schools. This renders the Gale–Shapley mechanism manipulable. P. Pathak and T. Sönmez (Pathak, Sönmez, 2013) were the first to compare the degree of manipulability of the constrained Boston and Gale–Shapley mechanisms in order to justify the switch from the former to the latter in Chicago in 2009. Their work was followed by a list of papers comparing those two and several other mechanisms from the manipulability perspective, presenting different notions of comparing manipulability (Bonkougou, Nesterov, 2021, 2023; Chen, Kesten, 2017). Most of the literature investigated the degree of those manipulability mechanisms' introduced in one or another way and very few researchers analyzed the actual effects of the manipulations.

This work presents the simulation-based comparative analysis on the harm of manipulations in the constrained Boston and Gale–Shapley school choice mechanisms. We work under the assumption that students know the priorities of schools and with the notion in which students are allowed to adjust their preferences during a certain period of time in response to the preference profile they observe. Among practical applications of this approach we can mention the Wake County Public School System (WCPSS), iterative student allocating mechanisms in Brazil, Germany, and the province of Inner Mongolia in China (Bo, Hakimov, 2022) and some others. The implementation of WCPSS in 2013–2014 was analyzed in (Dur, Hammond, Morrill, 2018). This work investigated the effect of manipulability of Boston mechanism using real data from WCPSS and showed that: 1) sophisticated students tend to increase their probabilities of getting a school by manipulations; 2) sincere students tend to get over-demanded schools more frequently due to sophisticated students' manipulations. It is important to mention that in their second hypothesis U. Dur, R. Hammond and T. Morrill (Dur, Hammond, Morrill, 2018) did not analyze the effect of manipulations on the positions of schools in sincere students' preference lists but focused only on the “demand” for the schools instead, i.e. they stated that students tend to get over-demanded schools more frequently, but did not analyze how high these schools are in the preference lists of students on average. The paper also analyzed a range of other hypotheses, however, these hypotheses were not directly related to our research. And we omit the details on them. Another analysis on the topic is the article by U. Dur, R. Hammond and O. Kesten (Dur, Hammond, Kesten, 2021). This work provided certain theoretical and experimental support to iterative implementation of the Boston mechanism over the Gale–Shapley mechanism and under their assumptions analyzed the implementation of WCPSS in 2014–2015 along with their theoretical analysis and laboratory experiments.

In this work we model the approach in the following way. Students submit their preferences during two periods of time. During the first period students do not observe any information on the others' preferences, and – what is important, – students are informed that during the second period they will be able to see the preference profile and will have the opportunity to change their preferences. This approach suggests that sophisticated students do not have any incentives to manipulate during the first period since they will get access to the information on the preference profile later, in the second period, and will be able to adjust their preferences if needed. While U. Dur, R. Hammond and T. Morrill (Dur, Hammond, Morrill, 2018), and other authors analyzed the real data, which means they had no direct access to the truth-telling preference

profile and, therefore, could work only with the actual profile, i.e. profile after manipulations, we can work both with the outcomes under truth telling and the corresponding outcomes after manipulations and analyze the effect of the manipulations directly. Our approach can be considered as an alternative to retrieving real preferences procedure used in literature, e.g. (Hernández-Chanto, 2021).

However, instead of observing the real data and trying to retrieve the truth-telling profile, we are constructing the truth-telling profiles as a big sample of problems and then trying to model the manipulations of students as realistically as possible. Our two-step mechanism design approach provides good support for the validity of our manipulations modeling. This approach allows us to significantly extend the analysis. For instance, we can now check whether sophisticated students increasing their probabilities of getting schools compared to sincere students (hypothesis 1 of (Dur, Hammond, Morrill, 2018)) means that they increase these probabilities while the probabilities of sincere students remain unchanged, which would be generally a positive effect of manipulations, or they do it at the expense of sincere students' probabilities of getting schools becoming lower. In this paper we show that we observe the latter.

We model students' behavior during the second period in the following way: sophisticated students, at some points in time, enter the system and adjust their preferences in a way that gives them the best possible school with respect to the current preference profile. And the preference profile is adjusted correspondingly. Some students enter the system only once, some – several times. These numbers of enters (so called “sophistication levels of students”) are generated randomly following the distribution similar to the one observed in WCPSS (2013–2014) with outliers dropped due to their low effect on the outcomes and significant effect on the computational time. Time slots for students entering the system are generated randomly after each sophisticated student is assigned his “sophistication level”.

Additionally, in section 5 of the paper we analyze the standard one-step mechanism design approach and compare its results with the results of the approach described above. Modeling of the behavior of sophisticated students for the one-step approach is also discussed in section 5.

The first paper to carry a comparative welfare analysis of the Boston and Gale–Shapley mechanisms was the article by A. Abdulkadiroğlu, Y. Che and Y. Yasuda (Abdulkadiroğlu, Che, Yasuda, 2011). This work supported the Boston mechanism based on the average welfare of students. In our research we make a similar analysis under different assumptions. We move from the assumption of students playing the game and reaching the Bayesian Nash equilibrium. Instead we are working with a two-step mechanism design approach and modeling the behavior of sophisticated students directly as the responses to the preference profile observed at each point in time. We also relax the assumption of all students sharing one common ordinal preference order over schools. Instead, we analyze two cases – in the first case students have arbitrary independently randomly generated preferences, in the second – students have correlated preferences. Details of the models and results are presented and discussed in sections 3 and 4.

2. Model

Formally, the standard model of the school choice problem which we use in this research can be described as follows. There is a finite set of students I with generic

element i , finite set of schools S with generic element s , and the vector of capacities of schools Q with generic element q . Each student has a strict preference relation P_i over $S \cup \{\emptyset\}$, where \emptyset represents the outside option. In this research \emptyset will represent the case of student being unassigned to any school by the mechanism and it is assumed that being unassigned is worse for any student than getting any school from set S . The set of preferences of all students $P = \{P_1, \dots, P_n\}$ is called a preference profile. Each school has a strict priority order $(f_s$ over I . The set of priorities of all schools $f = \{f_1, \dots, f_m\}$ is called a priority profile.

Formally the school choice mechanism is a function with the tuple $\{I, S, Q, P, f\}$ as an input and an allocation of students to schools as an output.

In this paper, only the constrained Boston and the constrained Gale–Shapley mechanisms are analyzed. Formally, they can be described in the following way.

The k -constrained Boston mechanism.

Step 1. Only the first choices of students are considered. For each school, consider the students who have listed it as their first choice and assign seats of the school to these students one at a time following their priority order until there are no seats left or there is no student left who listed it as her first choice.

Step r . Consider the remaining students and schools at position r in their preferences. For each school consider the students who listed it as their top r choice and assign the remaining seats to these students one at a time following their priority order until there are no seats left or there is no student left who listed it as her top r choice.

The mechanism stops after step k or earlier if all students are assigned or all seats are allocated at any earlier step.

The k -constrained Gale–Shapley mechanism.

Step 1. Only the first choices of students are considered. For each school, consider the students who have listed it as their first choice and assign seats of the school to these students one at a time following their priority order until there are no seats left or there is no student left who has listed it as her first choice (same as in Boston).

Step r . Each student who was rejected at step $r-1$ applies to his next highest choice. Each school considers these students and students who are temporarily held from the previous step together and rejects the lowest-ranking students in excess of its capacity, keeping the rest of students temporarily (so students not rejected at this step may be rejected at later steps.)

The mechanism stops if either all students are assigned or if all unassigned students already tried to apply to all k schools in their lists. Note that the k -constrained Gale–Shapley mechanism is not necessarily finishing in up to k steps.

3. Uncorrelated (random) preferences

We start our analysis in the environment of random preferences. In this research we fix the total number of students and the number of schools: $n = 100$, $m = 7$. Then we consider two “scenarios of priorities”, i.e. whether schools have the same or different priorities and four sets of parameters: $\{k = 2, n_{soph} = 15\}$, $\{k = 2, n_{soph} = 30\}$, $\{k = 4, n_{soph} = 15\}$ and $\{k = 2, n_{soph} = 30\}$. The results are presented in the tables, and each table corresponds to a certain “scenario of priorities” and contains information on all four sets of parameters. Further details on the data generation differ between uncorrelated and correlated preferences environments and are discussed in sections 3.1 and 4.1 respectively.

3.1. Data generation

For the uncorrelated preferences environment the data generation process was as follows. For each table and fixed set of parameters 32 cardinal preference profiles were randomly generated and for each cardinal preference profile 100 random problems were generated. By problem we mean the following: a random ordinal preference profile, a random priority profile of schools (accounting for the “scenario of priorities”), time slots assigned to sophisticated students and the capacities of schools. Students were assigned cardinal utilities on top of their ordinal preferences in order to estimate welfares of the mechanisms and estimate the effects of manipulations on average welfares of students as well as to compare two mechanisms in terms of average welfares they provide. Randomly generated cardinal preferences approach goes in line with the idea of different students having different “gaps” in their estimations of suitability of different schools. Cardinal utilities were generated as random numbers from 0 to 1 and sorted in increasing order.

Table 1 illustrates the idea for 6 students and 3 schools. Ordinal preference profiles and priority profiles were generated randomly, independently and uniformly (for each problem – 7 independently randomly generated priorities of schools for the “arbitrary priorities” scenario or 1 common randomly generated priority order of schools for the “common priority” scenario). Time slots were generated as random numbers from 0 to 1 and the order of students entering the system coincided with the order of the generated time slots. Each time a student entered the system he adjusted his preferences in a way to maximize his utility with respect to the current preference profile (and the preference profile was adjusted correspondingly). This was modelled as an immediate action after which student immediately left the system potentially re-entering it on a later step if he was sophisticated enough.

Finally, regarding the capacities – firstly, the total capacity (Q_{total}) was generated as a random number for each problem, uniformly distributed from 70 to 110. After that, each school was assigned 7 seats. And at the last step, the remaining ($Q_{total} - 49$) seats were uniformly randomly distributed across all schools. This approach led to a reasonable distribution of seats over schools. Dividing the sample into 32 cardinal profiles and 100 problems for each profile instead of simply generating 3200 problems, where cardinal profiles would be also random for each problem, allows for the robustness

Table 1.

Illustration of a random realization of cardinal preferences for 6 students and 3 schools

Student	Third-best school utility	Second-best school utility	Top school utility
1	0.09688441	0.23275192	0.54366176
2	0.13997268	0.14010473	0.55781963
3	0.26657585	0.70570927	0.71465156
4	0.07376943	0.19179078	0.89646396
5	0.02912025	0.06069377	0.64046142
6	0.51190481	0.69344622	0.82217131

evaluations of the results. We calculate standard deviations of the results with respect to different cardinal utility profiles generally observing reasonably low variances. This in turn allows us to make general conclusions based on the results without the risk of presenting certain random effects as findings of the research.

3.2. Results

We present the results in the form of the tables with the following structure. Four columns of the table correspond to four sets of parameters (values of k and n_{soph}). The first 6 rows present the share (percentages) of students getting schools. The next 6 rows present the average positions (ranks) of schools in the ordinal preferences of students in the outcome⁴. The last 6 rows present the average cardinal utilities of students. For each of those 6 rows sets the first two rows provide the information on all students, the next two rows – on sophisticated students only and the last two rows – on sincere students only. Finally, the odd rows show the results for the truth-telling preference profile, while the even rows – the results for the profile after manipulations. Cells in the tables are divided into two – the left one is for the Gale–Shapley mechanism and the right one is for the Boston mechanism. Mean values of the results are in the top part of the cells and the corresponding standard deviations are below in the brackets. The most significant differences (both from the perspective of values themselves and the importance of the differences) between the two mechanisms are highlighted in bold. This procedure was only implemented for the first and the third block of the results due to significant differences between mechanisms for the second block of the results for the vast majority of the scenarios under consideration. All further tables share this structure.

We start the analysis with the scenario of schools having arbitrary independent priorities. Table 2 presents the results.

Table 2 gives certain insights on the connections between parameters of the problems and the outcomes, as well as on the differences between two mechanisms under consideration. Firstly, it can be clearly seen that sophisticated students indeed increase their probabilities of getting schools by manipulations, which supports the hypothesis in (Dur, Hammond, Morrill, 2018). However, since these authors work with real data, the truth-telling profile cannot be observed in their research, while in this research our data generation process and the assumption of students submitting their true preferences during the first period due to the two-period mechanism design approach, give us the opportunity to analyze the effects of the manipulations from the perspective of comparing outcomes under truth-telling and under manipulations. This analysis shows that while sophisticated students increase their probabilities of getting schools by manipulations, they do it at the expense of sincere students' chances of getting schools becoming lower. This result is not trivial, since it potentially could be the case that sophisticated students were simply filling the empty seats left under the truth-telling outcome without causing a negative effect on the sincere students.

It is also worth mentioning that while U. Dur, R. Hammond and T. Morrill (Dur, Hammond, Morrill, 2018) work only with the Boston mechanism, our analysis is also provided for Gale–Shapley mechanism, and the effects of manipulations on both mechanisms regarding the probabilities of sophisticated and sincere students getting schools have similar patterns as discussed above. The differences in the changes of percentage

⁴ Schools are numbered starting from 0, so top 0 school is the best school, top 1 school is the second best etc. Also, students not getting any school are not accounted for in that metric.

Table 2.

Arbitrary school priorities

Metric	$n_{soph} = 15, k = 2$		$n_{soph} = 30, k = 2$		$n_{soph} = 15, k = 4$		$n_{soph} = 30, k = 4$	
	GS	Boston	GS	Boston	GS	Boston	GS	Boston
Students getting a school, %	87.47 (0.734)	86.06 (0.668)	87.50 (0.895)	86.09 (0.837)	88.69 (0.879)	88.20 (0.829)	88.91 (1.494)	88.38 (1.443)
Students getting a school (after manipulation), %	87.67 (0.748)	86.33 (0.682)	87.88 (0.924)	86.63 (0.86)	88.70 (0.878)	88.23 (0.825)	88.93 (1.493)	88.43 (1.432)
Sophisticated students getting a school, %	87.32 (1.01)	86.04 (1.05)	87.66 (1.064)	86.07 (1.063)	88.76 (1.284)	88.32 (1.151)	88.90 (1.699)	88.39 (1.496)
Sophisticated students getting a school (after manipulation), %	98.99 (0.233)	99.36 (0.221)	98.02 (0.325)	98.38 (0.227)	95.93 (0.746)	99.39 (0.244)	95.13 (1.03)	98.48 (0.419)
Sincere students getting a school, %	87.49 (0.783)	86.07 (0.707)	87.43 (0.879)	86.09 (0.831)	88.68 (0.85)	88.18 (0.84)	88.92 (1.495)	88.37 (1.536)
Sincere students getting a school (after manipulation), %	85.68 (0.853)	84.03 (0.799)	83.53 (1.238)	81.59 (1.168)	86.26 (0.956)	81.59 (0.965)	86.27 (1.836)	84.13 (1.977)
Average school rank of students	0.243 (0.009)	0.047 (0.002)	0.241 (0.009)	0.047 (0.003)	0.722 (0.032)	0.096 (0.006)	0.719 (0.053)	0.098 (0.014)
Average school rank of students (after manipulation)	0.313 (0.012)	0.082 (0.003)	0.39 (0.022)	0.127 (0.006)	0.798 (0.039)	0.125 (0.005)	0.875 (0.07)	0.164 (0.015)
Average school rank of sophisticated students	0.242 (0.018)	0.046 (0.007)	0.242 (0.012)	0.046 (0.005)	0.725 (0.034)	0.099 (0.012)	0.71 (0.059)	0.096 (0.019)
Average school rank of sophisticated students (after manipulation)	0.58 (0.034)	0.245 (0.017)	0.621 (0.044)	0.272 (0.018)	1.079 (0.065)	0.246 (0.018)	1.099 (0.104)	0.272 (0.033)
Average school rank of sincere students	0.243 (0.01)	0.046 (0.002)	0.241 (0.009)	0.047 (0.003)	0.721 (0.034)	0.095 (0.006)	0.722 (0.054)	0.098 (0.015)
Average school rank of sincere students (after manipulation)	0.253 (0.009)	0.045 (0.003)	0.259 (0.01)	0.043 (0.004)	0.74 (0.036)	0.096 (0.007)	0.76 (0.055)	0.098 (0.015)
Average utility of all students	0.742 (0.012)	0.75 (0.011)	0.741 (0.013)	0.749 (0.012)	0.741 (0.014)	0.758 (0.012)	0.702 (0.02)	0.762 (0.014)
Average utility of all students (after manipulation)	0.737 (0.013)	0.749 (0.011)	0.729 (0.014)	0.746 (0.012)	0.691 (0.014)	0.755 (0.012)	0.686 (0.021)	0.756 (0.015)
Average utility of sophisticated students	0.737 (0.024)	0.747 (0.024)	0.745 (0.022)	0.752 (0.02)	0.696 (0.03)	0.758 (0.026)	0.702 (0.026)	0.763 (0.021)
Average utility of sophisticated students (after manipulation)	0.793 (0.024)	0.838 (0.025)	0.786 (0.023)	0.831 (0.02)	0.709 (0.031)	0.834 (0.029)	0.705 (0.027)	0.829 (0.02)
Average utility of sincere students	0.743 (0.012)	0.751 (0.012)	0.739 (0.014)	0.748 (0.013)	0.699 (0.015)	0.758 (0.013)	0.702 (0.021)	0.761 (0.017)
Average utility of sincere students (after manipulation)	0.727 (0.013)	0.733 (0.012)	0.705 (0.016)	0.709 (0.015)	0.687 (0.015)	0.741 (0.013)	0.678 (0.023)	0.724 (0.019)

points (increase for sophisticated students and decrease for sincere students) are mainly observed because of the difference in the total numbers of sophisticated and sincere students. Thus, while we observe an increase in the percentages of sophisticated students getting schools due to manipulations and a corresponding decrease in the percentages of sincere students getting schools, the first two rows of the table suggest that the overall number of students getting schools is not significantly affected by manipulations.

Comparing two mechanisms we can state that, independently of the manipulations, the Gale–Shapley mechanism allocates, on average, more students to schools, however the difference is very small. Under manipulations this effect concentrates on the differences in sincere students' outcomes, while the opposite – a higher percentage of sophisticated students get schools under the Boston mechanism compared to the Gale–Shapley mechanism. Finally, regarding the parameter k it can be highlighted that increase in k leads to the statistically significant but relatively low average increase in the percentages of students getting schools. And regarding the parameter n_{soph} the analysis slightly complicates – for instance, for $k = 2$ after manipulations for both mechanisms it can be observed that the percentage of students getting schools increases with the increase in the number of sophisticated students. However, at the same time the percentage of sophisticated students getting schools decreases and the percentage of sincere students getting schools decreases. This might look like a paradox, however, it is explained by the change in the distribution of students into sophisticated and sincere ones.

The next block of results shows the average positions of schools in the preference lists that students actually get. Here even before the manipulations, a major difference between the Gale–Shapley and the Boston mechanisms can be seen – under the Boston mechanism students get schools much closer to their top choices on average compared to the Gale–Shapley mechanism. However, as was already mentioned, it is balanced by the lower percentages of students getting schools under the Boston mechanism. Whether one effect outweighs another will be discussed in the last block of results. It could be expected that the average positions of schools should increase after manipulations, i.e. become worse, due to sophisticated students who were not getting any schools under truth-telling, now getting certain schools possibly lower on their preference lists. It is important here to highlight again that students who were not getting any school were not accounted for in that metric.

Results show that indeed sophisticated students on average get significantly worse schools under manipulations. Another important result is that in terms of average positions of schools the manipulations do not cause any significant effect on the sincere students under both mechanisms. Finally, regarding the parameters k and n_{soph} the following observation can be made – while the number of sophisticated students does not affect the outcomes significantly, the increase in k , i.e. softening the constraint, expectedly leads to worse average schools for both mechanisms independently of manipulations, again by allocating more students to schools on average.

The final block of the results analyzes the average cardinal outcomes of students. The first important observation that should be made is that under the truth-telling, students get higher average cardinal utilities under the Boston mechanism compared to the Gale–Shapley mechanism, especially for the softer constraint k . This answers the question on the balance between bigger percentages of students getting schools under the Gale–Shapley mechanism and better average positions of schools under the Boston mechanism. This relation holds under manipulations as well.

The next important observation to be made – the effect of manipulations on the average cardinal outcomes of all students is barely statistically significant for the Gale–Shapley mechanism for 30 sophisticated students and insignificant for all other sets of parameters. However, as could be expected, the sophisticated students significantly increase their average cardinal outcomes by manipulations leading to the aver-

age decrease in the cardinal outcomes of sincere students. While the increase in the outcomes of sophisticated students is higher under the Boston mechanism, there is no difference in the decrease of the average outcomes of sincere students between the two mechanisms. Moreover, the higher average cardinal utilities observed for the Boston mechanism under truth-telling and statistically equal changes of the utilities for the sincere students under two mechanisms, together, result in higher average cardinal utilities of sincere students under the Boston mechanism compared to the Gale–Shapley mechanism even after manipulations.

However, the difference is statistically significant only for the softer constraint. Regarding k and n_{soph} – again no statistically significant effect on the number of sophisticated students is observed, however, one more interesting result can be observed regarding the constraint parameter. With the increase in k the average cardinal utilities of students decrease under the Gale–Shapley mechanism. This result could be, to some extent, counterintuitive. Indeed, we “give” students more options and opportunities and we could expect better average outcomes.

However, what seems to be observed is: while with softer constraint students indeed get more opportunities to get to some schools, this leads to worse average ordinal outcomes. While certain students get these worse schools it is done at the expense of other students losing those schools, which were higher in their preference lists than in the preference lists of students getting there now. Together with an insignificant or relatively small increase in the percentages of students getting schools, this indeed leads to a negative overall effect on the average cardinal utilities.

Table 3 provides the results for the scenario, in which all schools share the same common priority order over students.

In this part of analysis we will mostly concentrate on the differences between the first and the second scenario and not point out the similarities. This approach will be used further. The most crucial difference that can be observed between schools having arbitrary priorities and one common priority is the difference in the effects of manipulations on both mechanisms. If schools share the same common priority over students and the demand in schools does not differ critically, which goes in-line with our modeling, then smaller effects of manipulations can be expected due to less opportunities of manipulation. Results support this hypothesis, however the differences are relatively small. Another difference that can be drawn from the results is the difference in the average cardinal outcomes for $k = 4$ under the Gale–Shapley mechanism. Independently of manipulations, the average cardinal outcome is higher for the common school priority scenario. This effect is driven by the decrease in the average positions of schools the students get, i.e. by the better average positions. This in turn can be explained by the following: if all schools share the same common priority over students, then after the first one or two steps of the algorithm, students who did not get any school so far would not be able to get any worse schools by taking seats from other students, since they are generally low in the priority of schools, otherwise they would already get a seat.

Another important property in school choice problems is stability. An outcome is called unstable if there is a student i who wants a better school and either this school has unallocated seats or there is a student j currently allocated a seat in this school being lower in priorities of that school than the student i . Student i is called a

Table 3.

Common school priority

Metric	$n_{soph} = 15, k = 2$		$n_{soph} = 30, k = 2$		$n_{soph} = 15, k = 4$		$n_{soph} = 30, k = 4$	
	GS	Boston	GS	Boston	GS	Boston	GS	Boston
Students getting a school, %	86.58 (0.817)	86.08 (0.785)	86.54 (0.79)	86.04 (0.768)	88.14 (0.849)	87.95 (0.84)	88.23 (1.555)	88.02 (1.544)
Students getting a school (after manipulation), %	87.07 (0.843)	86.45 (0.798)	87.41 (0.82)	86.78 (0.757)	88.2 (0.851)	87.99 (0.836)	88.33 (1.551)	88.09 (1.547)
Sophisticated students getting a school, %	86.57 (1.227)	86.05 (1.208)	86.57 (0.834)	86.18 (0.839)	88.12 (1.232)	87.78 (1.105)	88.58 (1.547)	88.04 (1.623)
Sophisticated students getting a school (after manipulation), %	94.22 (1.14)	97.93 (0.488)	92.76 (0.697)	95.95 (0.491)	90.10 (1.182)	97.86 (0.444)	90.05 (1.513)	96.30 (0.953)
Sincere students getting a school, %	86.58 (0.803)	86.08 (0.774)	86.52 (0.832)	85.98 (0.82)	88.15 (0.826)	87.98 (0.848)	88.08 (1.646)	88.02 (1.631)
Sincere students getting a school (after manipulation), %	85.81 (0.844)	84.42 (0.881)	85.11 (0.953)	82.85 (1.001)	87.87 (0.847)	86.25 (0.944)	87.59 (1.678)	84.58 (1.963)
Average school rank of students	0.107 (0.004)	0.048 (0.004)	0.108 (0.006)	0.049 (0.003)	0.221 (0.007)	0.096 (0.006)	0.225 (0.015)	0.097 (0.011)
Average school rank of students (after manipulation)	0.149 (0.006)	0.09 (0.006)	0.183 (0.01)	0.131 (0.009)	0.235 (0.008)	0.131 (0.005)	0.249 (0.017)	0.164 (0.014)
Average school rank of sophisticated students	0.106 (0.011)	0.047 (0.007)	0.108 (0.007)	0.049 (0.005)	0.225 (0.016)	0.097 (0.013)	0.227 (0.022)	0.097 (0.016)
Average school rank of sophisticated students (after manipulation)	0.376 (0.029)	0.318 (0.029)	0.352 (0.023)	0.312 (0.023)	0.337 (0.022)	0.329 (0.022)	0.321 (0.032)	0.319 (0.038)
Average school rank of sincere students	0.107 (0.004)	0.048 (0.004)	0.108 (0.007)	0.048 (0.003)	0.221 (0.007)	0.096 (0.006)	0.225 (0.016)	0.097 (0.012)
Average school rank of sincere students (after manipulation)	0.104 (0.004)	0.041 (0.004)	0.103 (0.006)	0.035 (0.003)	0.217 (0.007)	0.088 (0.007)	0.218 (0.017)	0.081 (0.012)
Average utility of all students	0.749 (0.01)	0.751 (0.01)	0.746 (0.011)	0.748 (0.01)	0.747 (0.012)	0.758 (0.011)	0.75 (0.017)	0.761 (0.016)
Average utility of all students (after manipulation)	0.749 (0.011)	0.75 (0.011)	0.746 (0.011)	0.748 (0.011)	0.746 (0.012)	0.755 (0.012)	0.748 (0.018)	0.755 (0.018)
Average utility of sophisticated students	0.751 (0.029)	0.752 (0.028)	0.747 (0.017)	0.75 (0.017)	0.746 (0.035)	0.757 (0.035)	0.753 (0.024)	0.761 (0.023)
Average utility of sophisticated students (after manipulation)	0.787 (0.029)	0.825 (0.029)	0.773 (0.016)	0.805 (0.016)	0.751 (0.035)	0.816 (0.035)	0.754 (0.024)	0.807 (0.022)
Average utility of sincere students	0.749 (0.011)	0.751 (0.011)	0.746 (0.012)	0.748 (0.013)	0.747 (0.01)	0.758 (0.01)	0.748 (0.018)	0.761 (0.018)
Average utility of sincere students (after manipulation)	0.743 (0.011)	0.737 (0.011)	0.735 (0.013)	0.722 (0.013)	0.745 (0.01)	0.744 (0.01)	0.745 (0.018)	0.733 (0.02)

blocking student. We estimated the shares (percentages) of blocking students for the outcomes after manipulations for each set of the parameters. For the common school priority environment for Gale–Shapley mechanism we observed 1.5–1.9% blocking students for the constraint $k = 4$, and 5.5–7.2% blocking students for the constraint $k = 2$. Corresponding percentages of blocking students for the Boston mechanism were 9.4–10.6% and 10.2–11.7% respectively.

Thus, the Gale–Shapley mechanism showed higher stability for all four sets of parameters, i.e. lower numbers of blocking students. Increase in the constraint k expectedly showed positive effect on stability, especially under Gale–Shapley mechanism and the increase in the number of sophisticated students also showed a positive effect on the outcome after manipulations, although the effect was less significant. The last effect can be explained by the following: if those additional sophisticated students get better schools due to being sophisticated, they stop forming blocking pairs. At the same time, if such a student does not just take a spare seat and “kicks” some other student, then that “kicked” student might form a new blocking pair, but if at least some of those “kicked” students will not have justified envy, which can be potentially the case, then the total number of blocking students decreases. The same patterns were observed for the arbitrary school priorities scenario, however, shares (percentages) of students changed – 6.7–12.1% of blocking students for the Gale–Shapley mechanism and 13.1–13.8% of blocking students for the Boston mechanism. Lower stability for this scenario can be explained by the bigger effect of the constraint on the mechanisms. For the common priority, if some student got a school it was less likely for other arbitrary student to have justified envy compared to the arbitrary priorities of schools⁵.

4. Correlated preferences

While our results so far show an unclear effect of the increase in k on the average outcomes (cases of positive, negative and statistically insignificant changes are all presented in the results) this could be explained by completely randomly generated preference profiles, while in real life applications of the majority of preference profiles have a rather significant correlation in the preferences. We address this issue in this section by generating correlated preferences of students and analyzing the differences in the results.

4.1. Data generation

We generate the preferences similar to the procedure used in (Chen, Sönmez, 2006; Hakimov, Kesten, 2018). Each school is assigned a certain utility, common for all students. Then students and schools are divided into three districts and additional utility is added to each school-student pair for students living in the districts of the corresponding schools, and finally, an additional random utility component is added to each student-school pair representing the diversity of tastes of students. Exact values are chosen in a way to make the cardinal utilities as close as possible to the first data construction approach to allow for the comparative analysis of the results. Thus, the cardinal utilities of the best schools on average are 0.875 and 0.873 in the first and second data construction approaches respectively (checked on 1000 preference profiles). Average values for the second best, third best etc. schools are also similarly close. This approach led to an average value of Kendall’s coefficient of concordance (Kendall’s W) equal to 0.675 in the correlated preferences (checked on 1000 preference profiles) compared to average 0.01 for completely random preferences. Since the data generation approach changed and, now, cardinal and ordinal utilities are generated at the same time the samples for the new data consist of 32 preference profiles (now cardinal and ordinal profiles are not separated). For each preference profile 100 problems are generated, where the parameters that are not fixed and change from one problem to

⁵ Tables with the estimated mean values and standard deviations of percentages of blocking students for each scenario here and further are presented in the Appendix, A2.

another, are the total capacity together with the distribution of seats over schools, priorities of schools and the time slots at which students enter the system.

4.2. Results

Our main expectation for this scenario was: the increase in k should have a positive effect on the outcome as a result of a bigger effect on the total number of students getting schools due to the over demanded schools and low k leading to many students simply not being able to get into any school under truth-telling. Tables 4–5 present the results.

Obviously, the percentages of students getting schools and the average cardinal utilities drastically drop if we move from completely random preferences to the correlated preferences environment. And at the same time, as expected, an increase in k now has a positive effect on the average outcomes, especially for the Boston mechanism. Also, expectedly, the average positions of schools are significantly worse for the correlated preferences environment as well due to several top schools being highly contested. This also leads to a part of the effect observed regarding the average cardi-

Table 4.

Correlated preferences, arbitrary school priorities

Metric	$n_{\text{spk}} = 15, k = 2$		$n_{\text{spk}} = 30, k = 2$		$n_{\text{spk}} = 15, k = 4$		$n_{\text{spk}} = 30, k = 4$	
	GS	Boston	GS	Boston	GS	Boston	GS	Boston
Students getting a school, %	56.53 (2.121)	55.59 (1.965)	56.53 (2.491)	55.60 (2.142)	65.73 (1.093)	65.38 (1.04)	65.81 (1.879)	65.47 (1.826)
Students getting a school (after manipulation), %	60.32 (1.779)	56.97 (1.985)	64.06 (1.579)	58.90 (1.954)	69.75 (0.978)	65.50 (1.047)	73.89 (1.377)	65.85 (1.83)
Sophisticated students getting a school, %	55.94 (4.162)	54.75 (5.023)	56.48 (3.186)	55.42 (3.94)	65.08 (2.835)	65.08 (6.241)	65.67 (2.714)	64.74 (4.182)
Sophisticated students getting a school (after manipulation), %	98.64 (0.384)	96.96 (0.758)	96.58 (0.605)	93.40 (1.11)	99.52 (0.304)	97.08 (0.873)	98.73 (0.44)	93.57 (1.262)
Sincere students getting a school, %	56.64 (2.135)	55.74 (2.1)	56.55 (2.821)	55.68 (2.744)	65.84 (1.126)	65.43 (1.594)	65.87 (1.956)	65.79 (2.491)
Sincere students getting a school (after manipulation), %	53.56 (2.092)	49.91 (2.301)	50.13 (2.322)	44.11 (2.714)	64.49 (1.13)	59.93 (1.225)	63.24 (1.922)	53.98 (2.381)
Average school rank of students	0.459 (0.031)	0.11 (0.045)	0.459 (0.026)	0.114 (0.046)	1.394 (0.031)	0.487 (0.058)	1.401 (0.043)	0.481 (0.059)
Average school rank of students (after manipulation)	0.739 (0.043)	0.3 (0.04)	1.016 (0.047)	0.535 (0.042)	1.625 (0.028)	0.602 (0.062)	1.836 (0.055)	0.757 (0.075)
Average school rank of sophisticated students	0.456 (0.053)	0.105 (0.076)	0.451 (0.039)	0.105 (0.061)	1.38 (0.086)	0.478 (0.129)	1.404 (0.06)	0.493 (0.112)
Average school rank of sophisticated students (after manipulation)	1.553 (0.116)	0.867 (0.098)	1.656 (0.084)	0.991 (0.085)	2.461 (0.065)	0.871 (0.103)	2.486 (0.092)	1.033 (0.108)
Average school rank of sincere students	0.46 (0.032)	0.109 (0.043)	0.461 (0.031)	0.115 (0.045)	1.396 (0.03)	0.488 (0.061)	1.4 (0.053)	0.475 (0.073)
Average school rank of sincere students (after manipulation)	0.465 (0.031)	0.096 (0.041)	0.467 (0.028)	0.096 (0.048)	1.388 (0.027)	0.516 (0.065)	1.384 (0.048)	0.522 (0.097)
Average utility of all students	0.453 (0.016)	0.468 (0.015)	0.453 (0.018)	0.469 (0.014)	0.457 (0.01)	0.521 (0.01)	0.455 (0.012)	0.519 (0.014)

End of Table 4.

Metric	$n_{soph} = 15, k = 2$		$n_{soph} = 30, k = 2$		$n_{soph} = 15, k = 4$		$n_{soph} = 30, k = 4$	
Average utility of all students (after manipulation)	0.464 (0.014)	0.468 (0.015)	0.471 (0.014)	0.468 (0.016)	0.465 (0.01)	0.513 (0.01)	0.471 (0.011)	0.502 (0.015)
Average utility of sophisticated students	0.451 (0.032)	0.464 (0.039)	0.452 (0.022)	0.466 (0.03)	0.454 (0.023)	0.519 (0.048)	0.455 (0.02)	0.513 (0.036)
Average utility of sophisticated students (after manipulation)	0.664 (0.023)	0.739 (0.023)	0.632 (0.014)	0.689 (0.021)	0.555 (0.018)	0.733 (0.024)	0.547 (0.014)	0.688 (0.02)
Average utility of sincere students	0.453 (0.017)	0.469 (0.017)	0.454 (0.021)	0.47 (0.021)	0.458 (0.01)	0.521 (0.012)	0.455 (0.013)	0.522 (0.019)
Average utility of sincere students (after manipulation)	0.428 (0.016)	0.421 (0.017)	0.402 (0.018)	0.373 (0.019)	0.449 (0.01)	0.475 (0.01)	0.439 (0.012)	0.423 (0.016)

Table 5.

Correlated preferences, common school priority

Metric	$n_{soph} = 15, k = 2$		$n_{soph} = 30, k = 2$		$n_{soph} = 15, k = 4$		$n_{soph} = 30, k = 4$	
Mechanism	GS	Boston	GS	Boston	GS	Boston	GS	Boston
Students getting a school, %	55.10 (1.495)	54.79 (1.463)	55.36 (1.691)	55.06 (1.634)	65.01 (0.917)	65.01 (1.096)	65.41 (1.967)	65.31 (1.982)
Students getting a school (after manipulation), %	60.18 (1.464)	57.06 (1.572)	65.88 (1.423)	61.40 (1.873)	69.31 (0.847)	65.44 (1.055)	74.19 (1.249)	66.74 (2.079)
Sophisticated students getting a school, %	54.45 (5.117)	54.49 (6.596)	55.15 (2.751)	54.77 (3.465)	64.69 (3.208)	65.27 (5.164)	64.96 (3.058)	64.27 (4.597)
Sophisticated students getting a school (after manipulation), %	98.69 (0.543)	96.05 (1.216)	97.02 (0.936)	93.29 (1.631)	99.54 (0.202)	96.42 (1.009)	98.89 (0.502)	92.37 (2.209)
Sincere students getting a school, %	55.23 (1.926)	54.84 (1.979)	55.45 (2.245)	55.18 (2.548)	65.07 (1.148)	64.96 (1.67)	65.60 (2.063)	65.75 (2.412)
Sincere students getting a school (after manipulation), %	53.39 (1.751)	50.18 (1.798)	52.54 (1.988)	47.73 (2.422)	63.97 (0.996)	59.97 (1.275)	63.61 (1.806)	55.76 (2.557)
Average school rank of students	0.255 (0.028)	0.119 (0.034)	0.247 (0.056)	0.107 (0.057)	0.797 (0.066)	0.475 (0.071)	0.81 (0.068)	0.482 (0.069)
Average school rank of students (after manipulation)	0.584 (0.037)	0.345 (0.044)	0.859 (0.041)	0.59 (0.058)	1.061 (0.063)	0.6 (0.066)	1.313 (0.074)	0.756 (0.072)
Average school rank of sophisticated students	0.235 (0.075)	0.102 (0.075)	0.239 (0.065)	0.097 (0.06)	0.786 (0.125)	0.475 (0.149)	0.82 (0.108)	0.504 (0.105)
Average school rank of sophisticated students (after manipulation)	1.595 (0.14)	1.083 (0.17)	1.635 (0.09)	1.195 (0.148)	2.075 (0.092)	1.077 (0.134)	2.117 (0.118)	1.231 (0.144)
Average school rank of sincere students	0.258 (0.028)	0.121 (0.035)	0.251 (0.059)	0.11 (0.061)	0.8 (0.068)	0.477 (0.072)	0.805 (0.074)	0.474 (0.078)
Average school rank of sincere students (after manipulation)	0.245 (0.028)	0.086 (0.031)	0.23 (0.054)	0.067 (0.041)	0.772 (0.068)	0.452 (0.079)	0.755 (0.072)	0.386 (0.091)
Average utility of all students	0.458 (0.011)	0.463 (0.01)	0.459 (0.013)	0.464 (0.014)	0.498 (0.008)	0.518 (0.008)	0.5 (0.014)	0.52 (0.015)
Average utility of all students (after manipulation)	0.476 (0.011)	0.468 (0.011)	0.497 (0.012)	0.483 (0.013)	0.508 (0.007)	0.513 (0.007)	0.521 (0.012)	0.511 (0.014)
Average utility of sophisticated students	0.453 (0.038)	0.46 (0.051)	0.457 (0.019)	0.462 (0.026)	0.495 (0.024)	0.52 (0.039)	0.496 (0.02)	0.51 (0.034)

End of Table 5.

Metric	$n_{soph} = 15, k = 2$		$n_{soph} = 30, k = 2$		$n_{soph} = 15, k = 4$		$n_{soph} = 30, k = 4$	
Average utility of sophisticated students (after manipulation)	0.652 (0.024)	0.701 (0.03)	0.635 (0.016)	0.663 (0.019)	0.597 (0.02)	0.698 (0.021)	0.591 (0.014)	0.655 (0.019)
Average utility of sincere students	0.459 (0.014)	0.464 (0.014)	0.459 (0.018)	0.465 (0.021)	0.499 (0.008)	0.518 (0.01)	0.503 (0.015)	0.524 (0.019)
Average utility of sincere students (after manipulation)	0.445 (0.012)	0.427 (0.013)	0.437 (0.017)	0.406 (0.02)	0.493 (0.007)	0.48 (0.008)	0.491 (0.014)	0.45 (0.019)

nal outcomes. It should be also pointed out that for the correlated preferences environment, the average percentages of sophisticated students getting schools decrease only under the Boston mechanism, while the expected significant drop in the average percentages of students getting schools is observed compared to the uncorrelated preferences environment.

Moreover, for the common priority of schools the percentages of sophisticated students getting schools for the Gale–Shapley mechanism increase and increase almost up to 100% from 90–92%. However, this does not lead to a higher share (percentage) of sincere students getting schools under the Boston mechanism. The Gale–Shapley mechanism still gives more seats on average to sincere students due to the significant advantage of this mechanism in terms of overall percentage of allocated seats. What is more important for the correlated preferences, this difference is big enough to outweigh the difference in the average positions of schools for sincere students. Thus, under correlated preferences, it is observed that sincere students get higher average cardinal utilities under the Gale–Shapley mechanism compared to the Boston mechanism, unlike in the random preferences environment with the only exception of $n_{soph} = 15$ and $k = 4$. For sophisticated students we observe the opposite picture from the comparison of the cardinal outcomes of the mechanisms. This leads to almost equal cardinal utilities of students overall under two considered mechanisms with the exception of the scenario of arbitrary priorities of schools and $k = 4$, where the Boston mechanism performs better. Other than that, comparisons of general increase in probabilities of sophisticated students getting schools by manipulations at the expense of sincere students getting schools with lower probabilities, worse average positions of schools for sophisticated students under manipulations (accounting only for those getting schools) etc. show relatively similar patterns to the results for randomly uniformly generated preferences of students.

Switch to the correlated preferences leading to lower percentages of allocated students on average also expectedly showed a significant effect on stability of the mechanisms. Thus, for the correlated preferences environment and common priority of schools we observed on average 25.9–30.4% (for $k = 4$) and 34.6–39.1% (for $k = 2$) of blocking students for the Gale–Shapley mechanism (bigger numbers again corresponding to lower numbers of sophisticated students) and 35.6–38.0% (for $k = 4$) and 39.1–42.2% (for $k = 2$) of blocking students for the Boston mechanism with the same pattern regarding the number of sophisticated students. And for the arbitrary priorities of schools – 26.2–40.2% of blocking students for the Gale–Shapley mechanism and 38.2–43.5% – for the Boston mechanism. Patterns regarding the parameters remained

unchanged and importantly for all problems average numbers of blocking students were higher for the Boston mechanism.

5. One-step mechanism design approach

In this section we analyze a one-step mechanism design approach, where all students submit their preferences only once, and strategize with respect to the assumption of submitted preferences coinciding with the real preferences. Formally, we introduce the information on the preference profile as follows: the real preference profile is fixed, all sincere students tell the truth independent of that profile, and each sophisticated student submits such a preference profile, that gives him the best possible school with respect to the submitted preference profile, coinciding with the real preference profile. This idea can be viewed from the following perspective – even if students realize that a certain number of other students will manipulate, they do not have information on whether a certain student is sincere or sophisticated, therefore, they have to make a certain assumption regarding the submitted preference profile even if they understand that it might differ from the real preference profile. So, the assumption that it coincides with the real preference profile is reasonable if the percentage of sophisticated students is not too high. In this analysis, we set this percentage either as 15% or as 30%, which helps us to maintain the aforementioned assumption reasonable.

Results show that the one-step approach gives sophisticated students a smaller advantage compared to the first approach since they get less accurate information regarding the preferences of others. This leads to lower average share (percentages) of sophisticated students getting schools in this approach and, correspondingly, higher average percentages of sincere students getting schools. However for a part of parameter sets the difference in two approaches on the percentages of sincere students getting schools was statistically insignificant. Due to the slightly lower percentage of sophisticated students getting schools under the one-step approach and nature of the manipulations, we expectedly observe slightly lower, i.e. better, average positions of schools that sophisticated students get. However, this does not have any statistically significant effect on the average positions of schools of sincere students. Concluding these results, we observe that the average cardinal utilities of students are statistically equivalent under two approaches. At the same time for certain parameters small differences in the average utilities of students can be observed if we analyze the outcomes of sincere and sophisticated students separately. However, due to rather small and not perfectly general differences both regarding the percentages of students getting schools and the average positions of schools, the majority of the differences in the cardinal utilities even for the separate analysis of sincere and sophisticated students are statistically insignificant. For the details of the results (see Appendix, A1, Tables A1–A4).

Finally, regarding the stability estimations – results for one-step approach resembled the results of the original approach with no statistically significant difference in any of the scenarios (all differences were within the standard deviations of the results).

6. Conclusion

Summing up the analysis, the following conclusions and practical suggestions could be made. Firstly, while the percentage of sophisticated students cannot be controlled directly, the mechanism of constraint parameter k , and increasing this parame-

ter as much as practically possible can be suggested from the average welfare perspective. Moreover, it can be suggested not only if we analyze all students together, but if we analyze the outcomes of sincere and sophisticated students separately as well. This suggestion also strengthens the theoretical results in school choice showing practical advantages of the softer constraints due to the higher vulnerability of mechanisms to manipulations for lower parameter k (Pathak, Sönmez, 2013; Bonkougou, Nesterov, 2021 etc.).

Also, softer constraint k leads to higher stability, i.e. lower percentage of blocking students, which is another advantage of making the constraint as high as practically possible. While as was mentioned the number of sophisticated students cannot be controlled directly, it can be still affected by certain policies. Our analysis can neither support nor caution policy makers from policies leading to higher numbers of sophisticated students based on our simulations. The only visible effect of this parameter was the slightly higher stability observed for higher numbers of sophisticated students. Moreover, this research is based on assumptions which remain relevant only for relatively low percentages of sophisticated students. If these percentages increase to at least half of the total number of students, it becomes more reasonable to analyze the problem from the game theoretical perspective. Such approach was used in (Pathak, Sönmez 2008). They refer to another work on the subject – by N. Ergin, T. Sönmez (Ergin, Sönmez, 2006) and state that “In a model where all students are sophisticated, the set of Nash equilibrium outcomes of the Boston game coincides with the set of stable matchings of the underlying economy” and “a transition to the student-optimal stable mechanism should be embraced by all student groups, for it would be in the best interest of all students.” At the same time they show that if there are both types of students – sincere and sophisticated ones, then “the Boston mechanism gives a clear advantage to sophisticated students, provided they can coordinate their strategies at a favorable equilibrium.” Thus, our research supports the findings from the perspective of sophisticated students having advantage compared to sincere students under Boston mechanism. At the same time by moving to the constrained mechanisms environment we show that the same problem appears in the Gale–Shapley mechanism as well. However, the problem remains more significant for the Boston mechanism for the constrained versions of the mechanisms.

No general univocal suggestion regarding the choice between the constrained Gale–Shapley and Boston mechanisms can be made solely on the presented analysis. While in the majority of scenarios the Gale–Shapley mechanism allocates higher percentage of students to school (all students and sincere students analyzed separately), students get worse schools on average under it, and the balance between these two effects and following this balance difference in the average cardinal utilities depends on the parameters and the environment of the problem. However, if we concentrate only on the results for the more realistic assumption of correlated preferences, then in the vast majority of cases Gale–Shapley provides higher average welfare to sincere students compared to the Boston mechanism. This result shows a significant difference from the results obtained in (Abdulkadiroğlu, Che, Yasuda, 2011).

At the same time, results regarding the average welfare of all students under two mechanisms are ambiguous and neither of the mechanisms shows a generally better performance. Due to the better results from the perspective of share (percentages)

of students getting schools and from the perspective of welfare of sincere students the Gale–Shapley mechanism can be suggested as more practically appealing in general. Moreover, stability estimations strengthen this conclusion – the Gale–Shapley mechanism shows higher stability than the Boston mechanism, i.e. smaller average numbers of blocking students, for all scenarios and all sets of parameters analyzed in the research. The difference varies from statistically insignificant for several cases up to 12–12.1 percentage points of blocking students differences for the arbitrary priorities of schools, correlated preferences and parameters $n_{soph} = 30$, $k = 4$, and 5–10 percentage points for several other cases. This result to some extent extends the S. Bonkougou, A. Nesterov (Bonkougou, Nesterov, 2020) analysis. In that paper they showed that for both types of school priorities – arbitrary priorities and one common priority, and under certain assumptions regarding the strategies of students – constrained Gale–Shapley and Boston mechanisms are not comparable from the perspective of counting blocking students. However, in their paper the comparison had to be rather strict – one mechanism had to always have weakly higher number of blocking students and in at least one scenario strictly higher than another mechanism for the formal comparison to be possible. In our research we estimate the average percentages of blocking students instead showing the advantage of the Gale–Shapley mechanism from the stability perspective.

Finally, this paper analyzes two approaches to run the mechanisms. And these two approaches do not show any general statistically significant differences in the results for the majority of the parameters and scenarios. In practice, a relatively large part of the information (regarding the parameters of the problem and the environment) can be known to school authorities in advance, which allows for the analysis to be more precise, which in turn can give the opportunity for authorities to adjust certain parameters before running any mechanism. If some certain other approaches are planned to be used in practice, additional analysis can be also suggested as adjustments to the approach can potentially have a significant impact on the results.

Modeling the behavior of students under incomplete information, in which students only observe historical data and have certain assumptions on the structure of the preference profile instead of actually knowing the profile or even assuming one particular profile requires an additional practical and theoretical analysis and is a subject of further research. Practical research including (Hakimov, Kesten, 2018) shows the presence of manipulability in real life applications even for theoretically strategy-proof mechanisms, which suggests more complicated and at the same time potentially less rational behavior of students and requires additional analysis, which will be a matter of further research as well.

APPENDIX

A1. Appendix A1 presents the results for the one-step mechanism design approach

Table A1.

One-step approach, arbitrary school priorities

Metric	$n_{soph} = 15, k = 2$		$n_{soph} = 30, k = 2$		$n_{soph} = 15, k = 4$		$n_{soph} = 30, k = 4$	
	GS	Boston	GS	Boston	GS	Boston	GS	Boston
Students getting a school, %	87.45 (0.866)	86.02 (0.773)	87.97 (0.969)	86.52 (0.877)	88.81 (0.817)	88.33 (0.798)	88.77 (0.955)	88.30 (0.925)
Students getting a school (after manipulation), %	87.64 (0.877)	86.27 (0.787)	88.35 (0.982)	86.99 (0.895)	88.82 (0.818)	88.36 (0.801)	88.78 (0.958)	88.35 (0.921)
Sophisticated students getting a school, %	87.38 (1.227)	85.98 (1.102)	87.87 (1.14)	86.6 (0.916)	88.78 (1.151)	88.32 (1.018)	88.90 (1.116)	88.38 (0.957)
Sophisticated students getting a school (after manipulation), %	97.77 (0.397)	97.86 (0.386)	96.09 (0.572)	95.93 (0.532)	95.29 (0.654)	98.12 (0.35)	94.05 (0.741)	96.12 (0.415)
Sincere students getting a school, %	87.46 (0.862)	86.03 (0.788)	88.02 (0.972)	86.49 (0.938)	88.82 (0.804)	88.33 (0.797)	88.71 (0.954)	88.27 (0.967)
Sincere students getting a school (after manipulation), %	85.85 (0.991)	84.22 (0.902)	85.03 (1.208)	83.16 (1.125)	87.68 (0.896)	86.63 (0.92)	86.52 (1.096)	85.01 (1.21)
Average school rank of students	0.244 (0.01)	0.047 (0.002)	0.237 (0.011)	0.049 (0.003)	0.712 (0.033)	0.097 (0.006)	0.718 (0.034)	0.097 (0.005)
Average school rank of students (after manipulation)	0.305 (0.014)	0.078 (0.004)	0.349 (0.02)	0.105 (0.004)	0.783 (0.038)	0.122 (0.005)	0.851 (0.043)	0.144 (0.006)
Average school rank of sophisticated students	0.242 (0.012)	0.047 (0.007)	0.235 (0.014)	0.048 (0.004)	0.712 (0.038)	0.097 (0.011)	0.715 (0.035)	0.096 (0.009)
Average school rank of sophisticated students (after manipulation)	0.546 (0.035)	0.225 (0.018)	0.529 (0.038)	0.215 (0.012)	1.042 (0.064)	0.225 (0.015)	1.049 (0.059)	0.227 (0.015)
Average school rank of sincere students	0.244 (0.01)	0.047 (0.003)	0.238 (0.011)	0.048 (0.003)	0.712 (0.034)	0.097 (0.006)	0.719 (0.035)	0.096 (0.005)
Average school rank of sincere students (after manipulation)	0.253 (0.011)	0.046 (0.003)	0.254 (0.012)	0.047 (0.004)	0.73 (0.035)	0.098 (0.006)	0.753 (0.036)	0.099 (0.007)
Average utility of all students	0.739 (0.012)	0.747 (0.011)	0.745 (0.011)	0.752 (0.01)	0.702 (0.014)	0.762 (0.011)	0.702 (0.013)	0.763 (0.011)
Average utility of all students (after manipulation)	0.735 (0.013)	0.746 (0.011)	0.737 (0.012)	0.751 (0.01)	0.695 (0.014)	0.76 (0.011)	0.689 (0.014)	0.759 (0.011)
Average utility of sophisticated students	0.742 (0.024)	0.75 (0.024)	0.742 (0.019)	0.75 (0.019)	0.704 (0.027)	0.762 (0.026)	0.704 (0.023)	0.765 (0.019)
Average utility of sophisticated students (after manipulation)	0.793 (0.024)	0.832 (0.025)	0.778 (0.02)	0.812 (0.019)	0.716 (0.028)	0.833 (0.027)	0.706 (0.023)	0.817 (0.019)
Average utility of sincere students	0.739 (0.012)	0.747 (0.012)	0.747 (0.013)	0.753 (0.012)	0.702 (0.014)	0.762 (0.011)	0.701 (0.013)	0.762 (0.012)

Table A2.

One-step approach, common school priority

Metric	$n_{soph} = 15, k = 2$		$n_{soph} = 30, k = 2$		$n_{soph} = 15, k = 4$		$n_{soph} = 30, k = 4$	
	GS	Boston	GS	Boston	GS	Boston	GS	Boston
Students getting a school, %	86.38 (1.017)	85.88 (1.002)	86.79 (0.902)	86.27 (0.871)	88.43 (1.03)	88.25 (1.021)	88.34 (1.002)	88.14 (0.984)
Students getting a school (after manipulation), %	86.86 (1.019)	86.22 (0.987)	87.62 (0.918)	86.91 (0.883)	88.50 (1.036)	88.27 (1.024)	88.46 (1.014)	88.21 (1.012)
Sophisticated students getting a school, %	86.23 (1.194)	85.80 (1.456)	86.70 (1.209)	86.22 (1.175)	88.53 (1.183)	88.13 (1.186)	88.37 (1.042)	88.18 (1.071)
Sophisticated students getting a school (after manipulation), %	93.71 (0.965)	96.41 (0.591)	92.48 (1.023)	94.19 (0.747)	90.51 (1.077)	96.68 (0.591)	89.90 (1.032)	94.21 (0.642)
Sincere students getting a school, %	86.41 (1.017)	85.90 (0.979)	86.82 (0.87)	86.29 (0.848)	88.41 (1.054)	88.27 (1.047)	88.33 (1.061)	88.12 (0.992)
Sincere students getting a school (after manipulation), %	85.65 (1.053)	84.43 (1.103)	85.54 (0.955)	83.79 (1.037)	88.15 (1.059)	86.79 (1.136)	87.84 (1.088)	85.63 (1.212)
Average school rank of students	0.108 (0.004)	0.047 (0.003)	0.106 (0.004)	0.047 (0.004)	0.219 (0.008)	0.097 (0.006)	0.218 (0.007)	0.096 (0.006)
Average school rank of students (after manipulation)	0.149 (0.007)	0.085 (0.004)	0.174 (0.008)	0.109 (0.005)	0.232 (0.009)	0.125 (0.006)	0.243 (0.01)	0.145 (0.005)
Average school rank of sophisticated students	0.108 (0.01)	0.047 (0.006)	0.106 (0.006)	0.047 (0.004)	0.219 (0.02)	0.097 (0.015)	0.218 (0.013)	0.097 (0.01)
Average school rank of sophisticated students (after manipulation)	0.374 (0.029)	0.292 (0.03)	0.33 (0.022)	0.251 (0.018)	0.326 (0.03)	0.285 (0.023)	0.316 (0.022)	0.26 (0.019)
Average school rank of sincere students	0.108 (0.004)	0.047 (0.003)	0.106 (0.005)	0.047 (0.004)	0.219 (0.009)	0.096 (0.007)	0.218 (0.008)	0.096 (0.006)
Average school rank of sincere students (after manipulation)	0.105 (0.004)	0.041 (0.003)	0.101 (0.005)	0.039 (0.004)	0.215 (0.008)	0.091 (0.007)	0.211 (0.008)	0.088 (0.008)
Average utility of all students	0.745 (0.013)	0.747 (0.013)	0.749 (0.011)	0.751 (0.01)	0.751 (0.014)	0.763 (0.014)	0.75 (0.012)	0.761 (0.011)
Average utility of all students (after manipulation)	0.745 (0.014)	0.746 (0.013)	0.749 (0.011)	0.75 (0.011)	0.751 (0.014)	0.76 (0.014)	0.749 (0.012)	0.757 (0.012)
Average utility of sophisticated students	0.742 (0.024)	0.745 (0.023)	0.747 (0.021)	0.749 (0.02)	0.749 (0.028)	0.758 (0.027)	0.751 (0.014)	0.762 (0.015)
Average utility of sophisticated students (after manipulation)	0.776 (0.026)	0.808 (0.026)	0.772 (0.02)	0.795 (0.019)	0.753 (0.028)	0.809 (0.029)	0.753 (0.014)	0.796 (0.015)
Average utility of sincere students	0.746 (0.015)	0.747 (0.014)	0.747 (0.011)	0.75 (0.011)	0.752 (0.015)	0.764 (0.015)	0.75 (0.015)	0.761 (0.014)
Average utility of sincere students (after manipulation)	0.739 (0.015)	0.735 (0.015)	0.74 (0.011)	0.731 (0.012)	0.75 (0.015)	0.751 (0.015)	0.747 (0.015)	0.74 (0.015)

Table A3.

Correlated preferences. One-step approach, arbitrary school priorities

Metric	$n_{soph} = 15, k = 2$		$n_{soph} = 30, k = 2$		$n_{soph} = 15, k = 4$		$n_{soph} = 30, k = 4$	
	GS	Boston	GS	Boston	GS	Boston	GS	Boston
Students getting a school, %	56.33 (1.591)	55.33 (1.442)	56.01 (1.665)	54.95 (1.439)	65.77 (1.289)	65.54 (1.258)	65.98 (1.598)	65.72 (1.557)
Students getting a school (after manipulation), %	59.71 (1.535)	56.33 (1.486)	62.12 (1.068)	57.00 (1.514)	69.49 (1.168)	65.45 (1.261)	73.34 (1.395)	65.62 (1.625)
Sophisticated students getting a school, %	56.10 (2.704)	55.40 (4.99)	56.33 (3.426)	55.02 (4.835)	66.03 (2.81)	67.09 (5.34)	65.96 (2.192)	65.84 (3.469)
Sophisticated students getting a school (after manipulation), %	94.83 (1.061)	90.22 (1.79)	89.51 (1.463)	82.45 (2.38)	98.09 (0.818)	91.22 (1.933)	96.10 (0.987)	83.87 (2.299)
Sincere students getting a school, %	56.38 (1.742)	55.32 (1.911)	55.87 (1.933)	54.92 (2.318)	65.72 (1.418)	65.26 (1.553)	65.99 (1.944)	65.67 (2.158)
Sincere students getting a school (after manipulation), %	53.52 (1.785)	50.35 (1.717)	50.38 (1.74)	46.10 (1.998)	64.44 (1.359)	60.90 (1.536)	63.58 (1.899)	57.80 (2.25)
Average school rank of students	0.459 (0.024)	0.116 (0.043)	0.468 (0.021)	0.123 (0.041)	1.404 (0.036)	0.499 (0.056)	1.396 (0.033)	0.483 (0.06)
Average school rank of students (after manipulation)	0.714 (0.027)	0.266 (0.041)	0.915 (0.045)	0.393 (0.047)	1.622 (0.031)	0.583 (0.056)	1.798 (0.036)	0.649 (0.057)
Average school rank of sophisticated students	0.449 (0.056)	0.103 (0.074)	0.475 (0.045)	0.131 (0.058)	1.4 (0.094)	0.494 (0.18)	1.403 (0.055)	0.501 (0.118)
Average school rank of sophisticated students (after manipulation)	1.489 (0.084)	0.755 (0.09)	1.484 (0.09)	0.759 (0.094)	2.423 (0.08)	0.752 (0.087)	2.417 (0.068)	0.782 (0.082)
Average school rank of sincere students	0.461 (0.028)	0.116 (0.042)	0.464 (0.024)	0.117 (0.041)	1.405 (0.043)	0.499 (0.068)	1.392 (0.036)	0.475 (0.078)
Average school rank of sincere students (after manipulation)	0.464 (0.027)	0.105 (0.04)	0.469 (0.022)	0.102 (0.041)	1.398 (0.041)	0.533 (0.076)	1.381 (0.033)	0.556 (0.085)
Average utility of all students	0.451 (0.013)	0.465 (0.013)	0.45 (0.011)	0.463 (0.01)	0.456 (0.014)	0.519 (0.011)	0.459 (0.012)	0.523 (0.013)
Average utility of all students (after manipulation)	0.46 (0.014)	0.464 (0.013)	0.467 (0.01)	0.464 (0.011)	0.463 (0.014)	0.513 (0.011)	0.473 (0.012)	0.51 (0.013)
Average utility of sophisticated students	0.452 (0.02)	0.47 (0.038)	0.454 (0.025)	0.464 (0.037)	0.456 (0.027)	0.527 (0.026)	0.458 (0.016)	0.524 (0.025)
Average utility of sophisticated students (after manipulation)	0.644 (0.02)	0.697 (0.023)	0.614 (0.018)	0.64 (0.022)	0.549 (0.028)	0.699 (0.027)	0.541 (0.015)	0.643 (0.023)
Average utility of sincere students	0.451 (0.015)	0.464 (0.017)	0.448 (0.013)	0.463 (0.017)	0.456 (0.014)	0.518 (0.011)	0.459 (0.015)	0.522 (0.017)
Average utility of sincere students (after manipulation)	0.428 (0.016)	0.423 (0.015)	0.404 (0.012)	0.389 (0.014)	0.447 (0.014)	0.48 (0.012)	0.443 (0.014)	0.453 (0.017)

Table A4.

Correlated preferences. One-step approach, common school priority

Metric	$n_{soph} = 15, k = 2$		$n_{soph} = 30, k = 2$		$n_{soph} = 15, k = 4$		$n_{soph} = 30, k = 4$	
	GS	Boston	GS	Boston	GS	Boston	GS	Boston
Students getting a school, %	55.64 (1.754)	55.30 (1.706)	55.29 (1.793)	54.85 (1.73)	65.31 (1.23)	65.20 (1.318)	65.16 (1.097)	65.2 (1.24)
Students getting a school (after manipulation), %	60.45 (1.341)	57.36 (1.7)	63.36 (1.609)	58.36 (1.93)	69.55 (1.196)	65.40 (1.283)	73.23 (0.938)	65.32 (1.168)
Sophisticated students getting a school, %	54.79 (4.547)	54.59 (5.277)	55.51 (3.116)	55.00 (3.78)	65.07 (3.027)	65.38 (7.138)	65.51 (2.239)	65.35 (3.406)
Sophisticated students getting a school (after manipulation), %	95.59 (1.902)	89.16 (2.888)	89.37 (2.483)	80.17 (3.42)	98.38 (1.073)	89.16 (2.88)	96.44 (1.039)	81.44 (2.301)
Sincere students getting a school, %	55.79 (1.904)	55.42 (2.16)	55.20 (2.061)	54.78 (2.091)	65.35 (1.368)	65.17 (1.675)	65.02 (1.237)	65.14 (2.075)
Sincere students getting a school (after manipulation), %	54.25 (1.689)	51.75 (1.96)	52.21 (1.86)	49.01 (1.963)	64.46 (1.365)	61.21 (1.52)	63.28 (1.206)	58.41 (1.693)
Average school rank of students	0.25 (0.05)	0.115 (0.049)	0.25 (0.043)	0.113 (0.044)	0.788 (0.068)	0.456 (0.075)	0.8 (0.075)	0.478 (0.074)
Average school rank of students (after manipulation)	0.548 (0.045)	0.301 (0.046)	0.737 (0.066)	0.407 (0.047)	1.048 (0.058)	0.548 (0.068)	1.263 (0.059)	0.624 (0.068)
Average school rank of sophisticated students	0.235 (0.076)	0.099 (0.074)	0.25 (0.06)	0.109 (0.054)	0.784 (0.138)	0.439 (0.191)	0.8 (0.114)	0.48 (0.116)
Average school rank of sophisticated students (after manipulation)	1.504 (0.141)	0.951 (0.177)	1.41 (0.131)	0.853 (0.115)	2.043 (0.092)	0.904 (0.146)	2.01 (0.07)	0.87 (0.134)
Average school rank of sincere students	0.252 (0.05)	0.117 (0.049)	0.25 (0.049)	0.114 (0.047)	0.789 (0.068)	0.461 (0.071)	0.8 (0.079)	0.48 (0.084)
Average school rank of sincere students (after manipulation)	0.243 (0.048)	0.097 (0.042)	0.231 (0.045)	0.769 (0.035)	0.45 (0.069)	0.091 (0.074)	0.755 (0.08)	0.462 (0.102)
Average utility of all students	0.46 (0.014)	0.465 (0.014)	0.456 (0.013)	0.461 (0.013)	0.499 (0.01)	0.519 (0.009)	0.498 (0.009)	0.518 (0.01)
Average utility of all students (after manipulation)	0.478 (0.013)	0.47 (0.015)	0.486 (0.012)	0.471 (0.014)	0.509 (0.01)	0.515 (0.01)	0.517 (0.008)	0.509 (0.009)
Average utility of sophisticated students	0.455 (0.035)	0.46 (0.042)	0.458 (0.02)	0.462 (0.027)	0.499 (0.023)	0.522 (0.049)	0.497 (0.015)	0.517 (0.025)
Average utility of sophisticated students (after manipulation)	0.641 (0.024)	0.663 (0.031)	0.609 (0.014)	0.603 (0.022)	0.592 (0.019)	0.666 (0.034)	0.584 (0.012)	0.609 (0.019)
Average utility of sincere students	0.461 (0.015)	0.466 (0.018)	0.456 (0.016)	0.461 (0.017)	0.499 (0.01)	0.518 (0.011)	0.498 (0.01)	0.518 (0.016)
Average utility of sincere students (after manipulation)	0.449 (0.014)	0.436 (0.017)	0.433 (0.015)	0.414 (0.017)	0.494 (0.01)	0.488 (0.011)	0.488 (0.009)	0.466 (0.012)

A2. Appendix A2 presents the stability estimations for all scenarios considered in the research. Numbers at the top of the cells show the average percentages of blocking students and numbers in the brackets show the corresponding standard deviations.

Table A5.

Percentages of blocking students for the original (several steps) approach, %

Metric	$n_{\text{oph}} = 15, k = 2$		$n_{\text{oph}} = 30, k = 2$		$n_{\text{oph}} = 15, k = 4$		$n_{\text{oph}} = 30, k = 4$	
	GS	Boston	GS	Boston	GS	Boston	GS	Boston
Arbitrary school priorities (Arbitrary preferences)	12.1 (1.0)	13.8 (1.0)	11.4 (0.9)	13.3 (0.9)	7.4 (0.6)	13.3 (0.9)	6.7 (0.5)	13.1 (0.9)
Arbitrary school priorities (Correlated preferences)	40.2 (1.9)	43.5 (1.9)	36.2 (1.2)	41.5 (1.4)	30.0 (1.0)	39.8 (1.3)	26.2 (1.0)	38.2 (1.0)
Common school priority (Arbitrary preferences)	7.2 (0.5)	11.7 (0.7)	5.5 (0.3)	10.2 (0.5)	1.9 (0.2)	10.6 (0.8)	1.5 (0.2)	9.4 (0.6)
Common school priority (Correlated preferences)	39.1 (1.4)	42.2 (1.6)	34.6 (1.3)	39.1 (1.7)	30.4 (1.0)	38.0 (1.5)	25.9 (1.2)	35.6 (1.6)

Table A6.

Percentages of blocking students for the one-step approach, %

Metric	$n_{\text{oph}} = 15, k = 2$		$n_{\text{oph}} = 30, k = 2$		$n_{\text{oph}} = 15, k = 4$		$n_{\text{oph}} = 30, k = 4$	
	GS	Boston	GS	Boston	GS	Boston	GS	Boston
Arbitrary school priorities (Arbitrary preferences)	12.1 (0.9)	13.7 (0.8)	11.6 (0.8)	13.4 (0.8)	7.6 (0.7)	13.4 (0.9)	6.9 (0.5)	13.2 (0.8)
Arbitrary school priorities (Correlated preferences)	40.1 (1.6)	43.4 (1.7)	36.6 (1.4)	41.8 (1.8)	30.4 (1.3)	40.2 (1.3)	26.3 (1.1)	38.4 (1.2)
Common school priority (Arbitrary preferences)	7.4 (0.4)	12.0 (0.6)	5.6 (0.3)	10.3 (0.7)	1.9 (0.2)	10.7 (0.6)	1.5 (0.2)	9.8 (0.7)
Common school priority (Correlated preferences)	39.6 (1.5)	42.9 (2.0)	35.0 (1.2)	39.8 (1.9)	30.6 (1.2)	38.2 (1.5)	26.3 (1.0)	36.3 (1.6)

REFERENCES / ЛИТЕРАТУРА

- Abdulkadiroğlu A., Che Y.K., Yasuda Y.** (2011). Resolving conflicting preferences in school choice: The “Boston mechanism” reconsidered. *American Economic Review*, 101 (1), 399–410.
- Abdulkadiroğlu A., Sönmez T.** (2003). School choice: A mechanism design approach. *American Economic Review*, 93 (3), 729–747.
- Bo I., Hakimov R.** (2022). The iterative deferred acceptance mechanism. *Games and Economic Behavior*, 135, 411–433.
- Bonkougou S., Nesterov A.** (2020). Reforms meet fairness concerns in school and college admissions. Basic research program. *Higher School of Economics Research Paper*. Working papers. WP BRP 234/EC/2020. Available at: <https://wp.hse.ru/data/2020/07/30/1598422634/234EC2020.pdf>
- Bonkougou S., Nesterov A.** (2021). Comparing school choice and college admissions mechanisms by their strategic accessibility. *Theoretical Economics*, 16, 881–909.
- Bonkougou S., Nesterov A.** (2023). Incentives in matching markets: Counting and comparing manipulating agents. *Theoretical Economics*, 18 (3), 965–991.

- Chen Y., Kesten O.** (2017). Chinese college admissions and school choice reforms: A theoretical analysis. *Journal of Political Economy*, 125 (1), 99–139.
- Chen Y., Sönmez T.** (2006). School choice: An experimental study. *Journal of Economic Theory*, 127 (1), 202–231.
- Dur U., Hammond R.G., Kesten O.** (2021). Sequential school choice: Theory and evidence from the field and lab. *Journal of Economic Theory*, 198, 105344.
- Dur U., Hammond R.G., Morrill T.** (2018). Identifying the harm of manipulable school-choice mechanisms. *American Economic Journal: Economic Policy*, 10 (1), 187–213.
- Ergin H., Sönmez T.** (2006). Games of school choice under the Boston mechanism. *Journal of Public Economics*, 90 (1–2), 215–237.
- Hakimov R., Kesten O.** (2018). The equitable top trading cycles mechanism for school choice. *International Economic Review*, 59 (4), 2219–2258.
- Hernández-Chanto A.** (2021). Recovering preferences in college assignment problems under strategic and constrained (truncated) reports. *SSRN Working Paper*, no. 3784310.
- Pathak P.A., Sönmez T.** (2008). Leveling the playing field: Sincere and sophisticated players in the Boston mechanism. *American Economic Review*, 98 (4), 1636–1652.
- Pathak P.A., Sönmez T.** (2013). School admissions reform in Chicago and England: Comparing mechanisms by their vulnerability to manipulation. *American Economic Review*, 103 (1), 80–106.

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О манипулируемости «ограниченных» Гейла–Шепли и Бостонского механизмов при распределении студентов по школам. Часть 2. Сравнительный анализ вреда манипуляций

Аннотация. В данной работе представлен сравнительный анализ вреда манипуляций в двух широко используемых механизмах выбора школы — ограниченном Бостонском механизме и ограниченном механизме Гейла–Шепли — с точки зрения доли поступающих, получающих места в школах, и среднего благосостояния поступающих. Таким образом, эта часть исследования расширяет анализ манипулируемости, представленный в первой части, но анализирует не саму возможность манипулирования, а вред манипуляций. Мы также анализируем связь между параметрами задач (процент «продвинутых» поступающих и ограничение механизма, т.е. максимальное число школ, которые поступающим разрешено указывать в своих предпочтениях) и результатами работы механизмов. Наконец, мы анализируем и сравниваем два различных дизайна механизмов: тот, в котором поступающие сообщают свои предпочтения в один и тот же момент времени, и тот, в котором поступающим разрешено менять свои предпочтения в течение определенного общего периода. В данной части исследования мы показываем следующие преимущества механизма Гейла–Шепли по сравнению с Бостонским: большая доля поступающих, получающих места в школах, и при реалистичном предположении о коррелированности предпочтений поступающих — более высокое среднее благосостояние «искренних» поступающих при среднем благосостоянии всех поступающих статистически одинаковым в двух механизмах в большинстве анализируемых сценариев.

Ключевые слова: *дизайн рынка, теория распределения студентов по школам, манипулируемость.*

Классификация JEL: C78, D47, D78.

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