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**Government regulation of the market for higher education**¹

**Abstract.** This paper presents a model of strategic competition between universities that accounts for the existence of positive spillover effect from education (peer effect). It was demonstrated that in the presence of peer effect strategic competition results in inefficient student allocation between the two universities (biased to the high-quality university) and excessive quality differentiation. The model is used to analyze the implications of government funding policies as well as admission and quality regulation. It was demonstrated that traditional schemes of institutional funding and students’ financial aid programs like tuition fee subsidy, quality investment subsidy, or total cost subsidy reduce social welfare. At the same time, an introduction of provision of tuition-free education for the best students combined with a per-student grant provided to the university improves both students’ and social welfare. It was also demonstrated that tight admission regulation is not socially desirable while the introduction of minimum quality standards makes society better off.

**Keywords:** higher education, strategic competition, peer effect, welfare.

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1. **Introduction**

Higher education institutions are usually regulated by the government. This is explained by efficiency and equity reasons. Individuals may understate the future impact of education on their wellbeing at the individual level and they definitely do not take into account the positive external effects that come from the impact of education on human capital accumulation and resulting economic growth. All these facts explain why higher education is quite often subsidized by the government.

The government contributes to financing of higher education by providing funds to institutions and also by providing tuition subsidies, grants and loans to the students. In spite of commercialization of higher education that results in introduction (or increase) of tuition fees (see the review of tuition fee reforms in OECD (OECD, 2015, p.304)), the public funding stays the dominant source of institutions’ finance in most of the countries. For example, in OECD countries public expenditure on tertiary educational institutions constitutes about 66% and varies widely across the countries from 25% in UK up to 94% in Austria (OECD, 2018, p. 277).

Since the higher education institutions (HEI) engage in strategic competition for the students, different funding policies may have different (and sometimes unanticipated) impact on institutions’ investment in education quality, tuition fees, and student enrollment.

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The government does not only provide a significant share of HEIs funding but also imposes some regulations. Taking into account that it might be difficult (or almost impossible) for individuals to differentiate between a low-quality university and a fake university\(^2\) that just imitates the education activity, the government imposes minimum quality standards to overcome this asymmetry of information. In most countries, this policy is implemented via the government accreditation of HEI (OECD, 2008, p. 263).

In addition to quality standards, the government quite often sets the minimum admission requirements. These requirements usually deal with the secondary school certificates and may in addition include some performance indicators like minimum grade point average (GPA) or minimum national examination scores. For example, in two-thirds of OECD economies, national examinations or standardized tests are required for admission to universities (OECD, 2017, p. 403).

Both government regulation (like quality and admission standards) and funding policy affect the outcome of strategic competition between universities, but only few studies could be found that investigate the implications of these policies / regulations. Moreover, most of the existing theoretical papers focus on different forms of tuition subsidies and loans (Epple, Romano, 1998; Fethke, 2005; Kemnitz, 2007; Del Rey, Racionero, 2010; Diris, Ooghe, 2018), while government funding in European economies is provided to institutions, not households. In this paper, a game-theoretic model of universities’ competition is provided and both government regulation and different forms of funding policies are analyzed within the same analytical framework. The analysis includes the institutions funding in the form of education cost subsidy or quality investment subsidy, students support via tuition fee subsidy, and a dual tuition system where universities provide tuition-free education for talented students in exchange for some fixed per-student subsidy and charge market price to other students (this scheme is used in some transition economies). Implications of the two forms of government regulation, quality standard and minimum admission standard, were also considered.

The remaining part of the paper is organized as follows. In section 2 the literature review is provided. In section 3 the theoretical model is outlined and in section 4 the equilibrium is derived and its efficiency analysis is undertaken. The policy analysis of different funding schemes as well as quality and admission regulation is presented in section 5. Section 6 concludes the paper.

2. Literature review

Governments regulate higher education and simultaneously contribute to its financing in most countries but few papers analyze the implications of government policy for strategic competition between universities. A lot of literature studies the problem of students’ matching to universities’ taking into the account students’ preferences (e.g., (Abdulkadiroglu, Sonmez, 2003)), but this question is beyond the scope of our analysis.

\(^2\) (Cohen, Winch, 2011) estimated that there were about 3000 fake universities in the world in 2011 with the growth rate of more than 40% between 2010 and 2011.
Most of the empirical papers are devoted to the analysis of US market of higher education and concentrate on the demand side since tuition fees represent the key source of universities’ revenue. These studies investigate the accessibility of higher education for low income families and minorities and the role of financial aid (Epple, Romano, Sieg, 2008; Epple et al., 2017). It was noted that an increase in state appropriation is accompanied by an increase in tuition fees (this relationship is known in the literature as ‘Bennett hypothesis’) and many papers tested the impact of financial aid on tuition fees (Singell, Stone 2007; Frederick et al., 2012). Tuition fees change represents only one implication of the policy; the others deal with the changes in enrollment and education quality. These issues are addressed by (Frederick et al., 2012), where, using simulation analysis, the implications of US Federal funding program known as «American Graduation Initiative» are analyzed. It was demonstrated that the policy implications crucially depend on whether the government funds are used to subsidize institutions or students. If the money is given in the form of students’ aid then it results in increased enrollment but reduced education quality. If, instead, the money is given to institutions, then it increases the quality of education but only slightly increases enrollment.

The papers that study the government funding and the government regulation simultaneously are especially rare. The most prominent paper in this field is (Berger, Kostal, 2002), where regulation and enrollment in the US public universities are analyzed. It was demonstrated that the reduced state appropriations under given regulation decrease public universities’ enrollment. But this effect might be partially offset if there is a proper combination of administrative and academic regulation. The authors explain this substitutability between the government funding and regulation by the fact that it prevents universities from reallocating revenues from teaching to research (under reduced state appropriation) in the presence of more rigid system of regulation.

In many European economies education at public universities is provided either for free or at very low tuition fees set by the government (OECD, 2018, p. 292), and the most part of the government spending on education in this case is represented by the institutional rather than student funding. The governments in European economies provide institutional funding as a combination of per-student subsidy and a fixed grant. The composition of this funding affects the universities’ choice between teaching and research. Using a model of competition between two state universities, where students differ both in terms of location and in terms of their ability, (Del Rey, 2001) demonstrates that, depending on the size of per student subsidy and the students ability, one can get one of the four possible equilibrium outcomes: the two extreme cases, where all the funds are allocated either for teaching or for research, and the two mixed cases, where either mass teaching is combined with some research or selective teaching (under rationed admission ) takes place with most of the funds allocated for research.
The result of (Del Rey, 2001) crucially depends on the modeling approach: the research funds are formed on a residual basis and, as a result, under a high per-student subsidy, a university finds it optimal to increase enrollment in order to increase research budget. If the total budget allocated to the university is fixed and does not depend on the number of students enrolled (De Fraja, Iossa, 2002), then education activity cannot be treated as a source of additional money for research but is undertaken to increase the university prestige. In this setup, a university cares about education quality, enrollment and research since it increases the university prestige. Assuming that students differ in transportation costs and ability and can study for free, the universities compete for the students by setting admission standards. It was demonstrated that if the mobility costs were not very high, then pure strategies equilibrium was necessarily asymmetric with one elite university that sets very high admission standard and admits the best students and the other university with low admission standard. The analysis in this paper is undertaken for the case of identical budget and neither the change in the overall budget nor the reallocation of the budget between the universities are considered.

Stratification of universities may be also observed under a mixed system that combines institutions funding under some allocated quota and paid tuition in excess of this quota. This funding scheme is quite popular in some transition economies. (Polishchuk, 2010) proposes a stylized model of this mixed system, where universities compete by setting admission standards for tuition-free enrollment and by setting tuition fees for enrollment in excess of this quota under the fixed total university capacity. It was demonstrated that the equilibrium may result in formation of elite university with high admission standard and mass education sector with low admission standard. By attracting the highly able students, the elite sector supports its collective reputation and the graduates receive a wage premium at the labor market. Since enrollment as well as the quality of education in this model is assumed to be exogenous, the change in state funding may affect only tuition fees and equilibrium wages.

Tuition-free education in combination with paid enrollment in excess of state-funded quota was also considered in (Fridman, Verbetsky, 2017). It was demonstrated that an increase in government funding under strategic universities competition do not necessarily result in welfare improvement even if the funding is efficiently allocated between the competing universities. The analysis in this paper is based on representative agent model and so the policy implications should be reconsidered for a more general setup.

In many European countries commercialization of higher education resulted in tuition fee deregulation. The possible implications of this by choosing the quality of education instead of curriculum choice. Inefficiency arises due to the absence of product differentiation: under identical tuition fees universities have no incentive to propose different quality levels since
higher quality that requires higher spending cannot be compensated by the higher revenue.

Marketization of higher education increases pressure for cost sharing that raises tuition fees. These changes are accompanied by some reallocation of government funds from the institutional funding to students’ financial aid. This financial aid can take the form of education vouchers that provide tuition-free education; pure loans, when students repay the cost of education irrespective of graduation success; graduate tax, where a loan is subsidized and the cost of subsidy is paid back by successful graduates; or income contingent loans, that is, loans that are repaid by successful graduates only. In several papers the authors analyzed different forms of students’ financial aid: (Epple, Romano, 1998) investigated the impact of vouchers for education at private schools; comparative analysis of different forms of loans can be found in (García-Peñalosa, Wälde, 2000; Kemnitz, 2007; Del Rey, Racionero, 2010).

Our paper combines the analysis of demand-side policies (tuition subsidies), supply-side policy (different forms of institutions’ subsidies) and government regulation (admission and quality standards) within the same analytical framework. This approach allows performing comparative analysis of demand and supply side policies that, to the best of our knowledge, is absent in the existing theoretical literature and also combines it with the analysis of direct government regulations. Moreover, in addition to purely demand-side or supply-side funding policies, this paper introduces a policy, which combines both sides simultaneously. This policy takes place when some students are admitted to the university on a free basis and the university gets a per-student grant (subsidy) to finance the education of those students while others pay tuition fees. This policy is widely used in many post-Soviet economies including Russia.

3. The model

The analysis is based on a vertical product differentiation model adopted for the analysis of higher education. The model of this sort was previously used in (Kemnitz, 2007; Eisenkopf, Wohlschlegel, 2012; Haupt et al., 2016).

Assume that there is a continuum of students of mass one who differ in ability $\theta$, that is uniformly distributed on the interval from $0$ to $b$. Each student obtains at most one unit of higher education. Thus, education brings some common benefit, or wage premium $\alpha$ for all students. In addition to this common term, each student gets individual benefit from education quality $q$ that depends on his own ability and is given by $\theta q$. This personalized component could be explained by wage premium and/or the individual costs of education.

Empirical studies (see the survey in (Epple, Romano, 2010)) indicate that student’s benefit may be also affected by the abilities of her/his classmates. This spillover effect is, called peer effect. Following classification
given by (Epple, Romano, 2010), the peer effect might be direct (when the presence of one student in the class affects the benefit of his classmate without changing anyone’s behavior) or indirect that works through the change in student/professor behavior (for example, when a professor changes the pattern of presentation due to high heterogeneity of the class or a student changes her/his efforts).

In the proposed paper we account for direct peer effect only. To incorporate this peer effect, quite standard modeling approach is used, where the benefit from education might be positively affected by the average ability of the students $\bar{\theta}_i$ enrolled in university $i$. It was introduced in (Arnott, Rowse, 1987) and later on used in many other papers (Epple, Romano, 1998; Epple et al., 2008).

We do not take into account indirect peer effect that may also affect the cost of teaching (see, for example, (Kemnitz, 2007)) and should account for the class heterogeneity in addition to the mean ability.

Denoting the tuition fee of a university $i$ by $p_i$, the consumer surplus of a student with the ability level $\theta$ from attending the university $i$ equals $\alpha + \beta \bar{\theta}_i + q_\theta (p - p_i)$, where coefficient $\beta$ ($\beta \geq 0$) reflects the intensity of the peer effect. The positive spillover effect given by $\beta \bar{\theta}_i$ may also come from the fact that the education is a status good, and prestige (status) of the university is higher if it can attract better students. Although $\beta \bar{\theta}_i$ allows different interpretations, for the sake of simplicity in subsequent analysis, this term will be further referred as the peer effect. So, we normalize to zero the consumer’s surplus of the student that does not acquire higher education.

Assume that not all students willing to acquire higher education will be able to do this due to the government regulation. Government imposes some minimum exam score that is required for the admission. Assume that this exam score is perfectly correlated with the student’s ability. Thus, only the students with $\theta \geq a$ will be allowed to get higher education. Finally, assume that those students that are allowed to get higher education are sufficiently differentiated, so that $b > 2a$.

Universities, both public and private, are usually considered as non-for-profit organizations. It is quite common to assume that the objective of a university is increasing in quality investment (Epple et al., 2006). Some researchers assume that universities care about such factors as number of students, average ability level of students and expenditures on research that represent a general form of ‘prestige’ (De Fraja, Iossa, 2002). Even though universities are taken as non-for-profit organizations, the objective function of a university could be given by maximization of net revenue from teaching. The motivation behind this statement is that universities maximize their research funds that are formed on residual basis. If the profit from teaching is used for research funding, then this illustrates the implementation of the idea of research fund maximization. This approach is used in (Eisenkopf, Wohlschlegel, 2012).

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3 Since the empirical evidence on peer effect is mixed (Epple, Romano, 2010), the possibility of zero peer effect intensity was included.

4 A dropout rate is not modeled explicitly, but it can be assumed that only low-ability students drop-out and the government introduces a minimum admission requirement in order to prevent those students from entering the university, assuming that the students cannot estimate correctly their graduation probability.
When it comes to the sources of higher education financing, it is usually assumed that funding comes from the tuition fees, government sources and endowment earnings (Del Rey, 2001, Epple et al., 2006). To find the net revenue from teaching, the cost function of a university should be specified.

Models of vertical product differentiation also differ in cost functions modelling: in some models, the costs are ignored at all (Shaked, Sutton, 1982) while in some papers quadratic quality production cost function is used (Moorthy, 1988). In some papers this quality production costs are treated as fixed costs (independent on the total output) while in others these costs are treated as variable (i.e. proportional to the output). In addition to the quality production costs, models may also include a fixed per unit costs, which is independent of quality investment.

Taking into account that government sets some minimum quality standards in education we assume that each university is required to spend at least some fixed amount $c$ per student. In addition to this required investment, the university may undertake additional quality investment that brings an additional spending of $\gamma (q_i)^2$ per student, where $\gamma > 0$ and $q_i$ is the quality of teaching of university $i$. This additional investment may include investment in sophisticated equipment, spending on improvement of teaching methods and attracting highly qualified professors. Thus, the university $i$ per student costs are given by $C_i = c + \gamma q_i^2$.

Assume that there are two universities that provide higher education in this economy. A two-stage game is considered, where initially the two universities simultaneously and independently decide on the quality investment levels. Then these levels become observable and at the second stage the two universities simultaneously set their tuition fees and finally students choose the university.

4. Equilibrium

Now let’s proceed to the analysis of equilibrium. As the game is sequential, the subgame perfect Nash equilibrium should be found. In the considered game, it could be derived via backward induction. Thus, the solution starts with the last stage, where under given quality levels the universities compete by choosing their tuition fees.

4.1. Price competition

When universities set their tuition fees, they anticipate the response of the students. Thus, we should start with the derivation of demands.

Without loss of generality we can assume that university 1 offers education of higher quality than university 2. Assuming that admission regulation is binding, the lowest type that will be admitted to university 2 is given by $a$. Now, we will find the marginal student $\hat{\theta}$ who is indifferent between universities 1 and 2: $\alpha + \hat{\theta} q_1 + \beta \bar{q}_1 - p_1 = \alpha + \hat{\theta} q_2 + \beta \bar{q}_2 - p_2$. Then all students with $\theta \in [\hat{\theta}, b]$ will attend university 1, and students with $\theta \in [a, \hat{\theta}]$ will attend university 2, which gives the following demands: $x_1(p) = b - \hat{\theta}$ and $x_2(p) = \hat{\theta} - a$. 

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Under the uniform distribution this implies the following average ability levels for the two universities: \( \bar{\theta}_i = 0.5(b + \bar{\theta}) \) and \( \bar{\theta}_2 = 0.5(\hat{\theta} + a) \). Plugging these expressions for the average ability levels into the marginal student equation and rearranging, we obtain:

\[
\hat{\theta} = \left(2(p_1 - p_2) - \beta(b - a)\right) / 2\Delta,
\]

where \( \Delta = q_1 - q_2 \) stays for the quality differential.

Given the demand functions derived above, the university \( i \) sets tuition fee that maximizes its net revenue \( R_i = (p_i - C_i)x_i(p_i, p_j) \). This objective function is strictly concave in the own price which implies that the first order condition is necessary and sufficient. The first order condition takes the form:

\[
p_i = C_i + x_i(p_i, p_j)\Delta,
\]

that is, the tuition fee has two components, the cost component and a strategic component that depends on the quality differential. An increase in quality differential increases the product differentiation that makes price competition less intensive and, as a result, allows some additional price markup.

Plugging (1) into (3) and solving the resulting system, we get the following prices:

\[
\begin{align*}
p_1 &= \left(4C_1 + 2C_2 + \beta(b - a) + 2\Delta(2b - a)\right) / 6; \\
p_2 &= \left(2C_1 + 4C_2 - \beta(b - a) + 2\Delta(b - 2a)\right) / 6,
\end{align*}
\]

that results in the following indifferent student type:

\[
\hat{\theta} = \frac{C_1 - C_2}{3\Delta} + \frac{b + a}{3} - \frac{\beta(b - a)}{6\Delta}.
\]

**4.2. Quality competition**

Now let’s proceed to the quality competition stage. By plugging the tuition fee (2) into the net revenue function and rearranging, the following expressions for the net revenue of university \( i \) are obtained: \( R_i = \Delta x_i^2 \). Thus, the university \( i \) chooses its quality by maximizing \( R_i \) given the demand specifications derived in section 4.1 subject to (4).

We will look at the interior equilibrium case only. Then the first order conditions are given by:

\[
\begin{align*}
\left(b - \hat{\theta}\right)^2 - 2\Delta(b - \hat{\theta})\left(\frac{\gamma}{3} + \frac{\beta(b - a)}{6\Delta^2}\right) &= 0; \\
-(\hat{\theta} - a)^2 + 2\Delta(\hat{\theta} - a)\left(\frac{\gamma}{3} - \frac{\beta(b - a)}{6\Delta^2}\right) &= 0.
\end{align*}
\]

As it is shown in the Appendix, if the peer effect is small, the second order conditions hold for both universities.

Solving the system, we obtain the equilibrium indifferent student type

\[
\hat{\theta} = \frac{b + a}{2} - \frac{4\beta\gamma}{9},
\]

the quality differential
\[ \Delta = \frac{3(b-a)}{4\gamma} \]  

(6)

and the quality levels for both universities

\[ q_1 = \frac{5b-a}{8\gamma} - \frac{\beta}{3}, \quad q_2 = \frac{5a-b}{8\gamma} - \frac{\beta}{3} \]  

(7)

This equilibrium is interior if \( \beta < 3(5a-b)/8\gamma \).

Plugging these values into the pricing functions (3) obtained at the previous stage, the equilibrium tuition fees are derived:

\[ p_1 = c + \frac{(5b-a)^2 + 24(b-a)^2}{64\gamma} \frac{\beta(b+3a)}{12} + \frac{\gamma\beta^2}{9}, \]  

(8)

\[ p_2 = c + \frac{(5a-b)^2 + 24(b-a)^2}{64\gamma} \frac{-\beta(3b+a)}{12} + \frac{\gamma\beta^2}{9}. \]  

(9)

Taking into account the marginal student type given by (5), it could be concluded that, in the presence of peer effect, the equilibrium enrollment is higher for the high-quality university: \( x_h = b-\check{\theta} = \frac{b-a}{2} + \frac{4\beta\gamma}{9} > \) \( b-a - \frac{4\beta\gamma}{9} = \check{\theta} = a = x_h \), which results in bigger research fund of high-quality university generated by the net revenue from teaching \( R_h = \Delta x_h^2 > \Delta x_l^2 = R_l \).

Plugging (7) and (5) into (10) and rearranging, we get the equilibrium value of total surplus:

\[ TS = (b-a) \left( \alpha-c + \frac{b^2 + 14ab + a^2}{64\gamma} \frac{\beta(a+b)}{2} + \frac{\beta^2\gamma}{27} \right). \]  

(10)

4.3. Equilibrium and efficiency

In order to find out the welfare implications of the strategic competition between the universities, the socially efficient allocation should be derived. Denoting by \( \check{\theta} \) the lowest ability student that is allocated to the high quality university, the total surplus is given by:

\[ TS(\check{\theta}) = \int_a^{\check{\theta}} (\alpha + \beta\theta^2 + q_1\theta - c - \gamma q_2^2) d\theta + \int_{\check{\theta}}^{b} (\alpha + \beta\theta^2 + q_1\theta - c - \gamma q_2^2) d\theta = \]  

\[ = (\alpha - c)(b-\check{\theta}) + \check{\theta} \frac{b^2 - a^2}{2} + q_2 \frac{(\check{\theta} - a)^2}{2} + q_1 \frac{\check{\theta}^2 - (b-\check{\theta})^2}{2} - \]  

\[ - \gamma q_2^2 \left( b-\check{\theta} \right) - \gamma q_2^2 \left( \check{\theta} - a \right). \]  

(11)

Maximizing (11) with respect to \( \check{\theta} \in [a,b] \), \( q_1 \geq 0 \) and \( q_2 \geq 0 \), the efficient allocation is derived, which is described in lemma 1.

**Lemma 1.** Socially efficient allocation requires positive and equal enrollment for both universities \( b-\check{\theta} = \check{\theta} - a = (b-a)/2 \), but different quality levels \( q_1^* = (3b+a)/8\gamma, \quad q_2^* = (b+3a)/8\gamma \), where students with \( \check{\theta} \in \left[ \check{\theta}, b \right] \) are allocated to the high-quality university and students with \( \check{\theta} \in \left[ a, \check{\theta} \right] \) are allocated to the low-quality one.
Proof: see Appendix B.

Now the equilibrium allocation derived in section 4.2 could be compared with the efficient one. The results of this comparison are summarized in proposition 1.

**Proposition 1. In the equilibrium:**

1) the low-quality university underinvests and high-quality university overinvests in the education quality in comparison with the socially efficient quality levels and as a result the equilibrium quality differential exceeds the socially efficient one;

2) in the presence of peer effect (if $\beta > 0$), the students’ allocation is inefficient: too many students are enrolled to the high quality university.

Proof: see Appendix.

The results of proposition 1 are quite standard for the models of vertical product differentiation. Universities use the quality investment strategically in order to reduce the intensity of price competition. Increased market power allows each university to raise the tuition fee and get positive net revenue from teaching. This incentive to reduce the intensity of price competition explains why the equilibrium differential exceeds the efficient one.

The peer effect distorts the allocation of the students between the two universities. Under given specification of preferences, even the students with lower quality preferences gain some benefit from studying with talented peers. Attracted by the better peer effect, the students with relatively low quality preferences choose the high-quality university. This misallocation reduces the peer effect benefits in both universities and results in efficiency loss.

4.4. Peer effect impact

In this section we analyze the implications of an increase in peer effect $\beta$ on the equilibrium and welfare.

**Proposition 2.**

An increase in peer effect intensity brings the following results: $\partial q_i / \partial \beta < 0$, $\partial p_i / \partial \beta < 0$ ($i = 1, 2$), $\Delta = 0$, $\partial(\; p_1 - p_2; / \partial \beta > 0$, $\partial x_i / \partial \beta = - \partial x_2 / \partial \beta > 0$, $\partial R_1 / \partial \beta > 0$, $\partial R_2 / \partial \beta < 0$, $\partial T S / \partial \beta > 0$.

Proof: see Appendix.

Under the given tuition fees and quality levels, an increase in peer effect intensity improves the strategic position of the high-quality university as it attracts better students and peer effect here is more pronounced. As a result, the higher quality university attracts more students and less students will be attracted by the low-quality university.

A stronger peer effect increases product differentiation and, as a result, reduces incentive for quality investment that is used as a strategic instrument of product differentiation. Reduced quality investment results in lower cost of education, which, in its turn, drives down the tuition fees. These changes in tuition fees are not symmetric: the high-quality university attracts better students and gains more from the increased peer effect intensity that increases its market power and, as a result, the tuition reduction for this university is smaller so that tuition differential goes up.
Finally, as the price-cost margin is explained by the strategic component of tuition fee (see (2)), which is proportional to the quality differential multiplied by enrollment and the quality differential stays the same, the price-cost margin changes in the same direction as the enrollment. Thus, the price-cost margin as well as the total net revenue increases for the high-quality university that attracts more students and is reduced for the low-quality one that suffers from the reduction of enrollment.

Although the peer effect distorts students’ allocation between the two universities as it was shown in proposition 1, the higher peer effect intensity improves social welfare. This happens because higher peer effect intensity reduces overinvestment in quality and this gain overweights the loss from the inefficient students allocation.

5. Policy analysis

5.1. Stricter minimum admission requirement set by the government

The government may affect strategic universities competition by setting the minimum admission requirement. In the model framework, the stricter minimum admission requirement is modeled via an increase in \(a\), that is, the lowest ability student admitted to the low-quality university.

**Proposition 3.**

An increase in minimum admission requirement bring the following results:

\[
\frac{\partial q_1}{\partial a} < 0, \quad \frac{\partial q_2}{\partial a} > 0, \quad \frac{\partial \Delta}{\partial a} < 0, \quad \frac{\partial x_i}{\partial a} < 0, \quad \frac{\partial p_i}{\partial a} < 0, \quad \frac{\partial R_i}{\partial a} < 0 (i = 1, 2), \quad \frac{\partial (p_i - p_1)}{\partial a} < 0, \quad \frac{\partial TS}{\partial a} < 0.
\]

**Proof:** see Appendix.

Let’s comment on the results of proposition 3. Increased minimum admission requirement has no direct impact for the high-quality university but reduces the enrollment of the low-quality university under given education qualities and tuition fees. Thus, the low-quality university is willing to restore its enrollment (at least partially) by attracting high-ability students, and to do this, it increases its quality investment. Increased minimum admission requirement reduces students’ diversity and as a result the large product differentiation becomes less profitable and the quality differential goes down.

Enrollment falls for both universities: the low-quality one suffers from reduced enrollment of the students with low exam scores and compensates this reduction partially by increasing quality investment and attracting some students with high ability, who under initial admission requirements, were enrolled to university 1. This explains the reduction of enrollment to university 1 as well.

For university 2, the cost component of tuition fee goes up due to the increased quality investment, but the strategic component goes down as the quality differential is smaller, which reduces the universities market power. In addition, less students are attracted, which reduces the university’s markup. As it is shown in proposition 3, strategic effect dominates cost effect for university 2 and its tuition fee goes down.
The situation is different for the high quality university. Reduced quality investment allows to economize on costs and this cost economy should reduce the tuition charged. In addition, the reduced quality differential decreases market power, and as a result reduces the university’s markup. Thus, its tuition fee goes down.

Reduced enrollment and increased costs (due to additional quality investment) are responsible for the reduction in net revenue of the low-quality university and increased tuition cannot fully compensate this reduction, so that the net revenue from education goes down. Although the high-quality university economizes on quality investment costs, the reduced enrollment results in lower net revenue for this university as well.

The universities definitely lose from this policy but the change in students’ welfare is ambiguous. Those students that lose the possibility to obtain higher education are definitely worse off. Enrolled students gain from lower tuition fees but their benefit from education also goes down due to the reduced quality investment.

The minimum admission requirement has two effects on the social welfare. On the one hand, it reduces excessive product differentiation by moving quality investment closer to the efficient levels but, on the other hand, it reduces total enrollment that lowers social welfare and the last effect dominates the first one.

5.2. Quality investment subsidy

It is well known that human capital investment might facilitate long run economic growth. As this positive external effect is not taken into account by the private agents, the government may subsidize higher education. These subsidies might be provided either to the universities or to the students.

Let us start the analysis of government subsidies with the subsidy provided to the higher education institution. The government may either subsidize the total cost of education or some components. First, the case of quality investment subsidy will be analyzed assuming that the same subsidy rate is applied for both universities’ quality investment. Denoting this subsidy rate by $s$, the subsidy-inclusive quality investment costs of university $i$ are given by $\gamma(1-s)(qi)^2$.

**Proposition 4.**

An increase in quality investment subsidy brings the following changes:
\[
\frac{\partial q_i}{\partial s} > 0, \quad \frac{\partial p_i}{\partial s} > 0 \quad (i=1,2), \quad \frac{\partial (p_1 - p_2)}{\partial s} > 0, \quad \frac{\partial \Delta}{\partial s} > 0, \quad \frac{\partial x_2}{\partial s} = -\frac{\partial x_1}{\partial s} > 0, \quad \frac{\partial R_2}{\partial s} > 0, \quad \frac{\partial R_1}{\partial s} \geq 0 \quad \text{if} \quad 9(b-a) / 8 \gamma(1-s) \geq \beta, \quad \frac{\partial TS}{\partial s} < 0.
\]

**Proof:** see Appendix.

Intuitively, the subsidy lowers the quality investment costs that makes such investment more profitable and as a result brings an increase in quality investment. Moreover, as the high-quality university invests more, it gains
more from this subsidy and as a result the increase in quality for the high-quality university is bigger, which increases the quality differential.

There are two effects that affect the equilibrium tuition fees. On the one hand, increased quality differential makes the products more differentiated and raises market power that increases tuition fees. On the other hand, the quality subsidy reduces the quality investment costs that reduce the costs of education and thus lowers tuition fees. But the last effect is partially mitigated by the increased quality investment so that the first effect becomes dominant and increased market power brings an increase in tuition fees.

The increased quality differential under given tuition fees should attract more students to university 1 but this positive impact is offset by a greater increase in tuition fee that makes university 1 less attractive. Finally, the second effect dominates and enrollment is reallocated in favor of university 2 in the presence of peer effect.

Increased tuition fee and reduced subsidy-inclusive costs increase per student net revenue for each university. This effect, combined with the increased enrollment, increases the net revenue for low-quality university. As the high-quality university enrollment goes down, the two effects have opposite impact on university 1 net revenue. Since the quality differential affects enrollment only due to the peer effect then if this peer effect intensity is small then the enrollment change is also small and as a result the effect of increased net per student revenue becomes dominant in this case, and the overall net revenue of high-quality university increases as a result of quality investment subsidy. But under high peer effect intensity, the fall of enrollment might overweight the per student revenue increase so that the total net revenue might fall.

Finally, we look at the change in social welfare. On the one hand, students’ allocation is improved as initially there were too many students enrolled to high-quality university and now the enrollment changes in favor of the low-quality university that improves social welfare. On the other hand, product differentiation increases, which reduces social welfare as differentiation was excessive even without subsidy. As it follows from proposition 4 the quality effect dominates the enrollment effect and thus total surplus is diminishing in subsidy rate.

5.3. Proportional cost subsidy
In addition to the quality investment costs, the universities incur constant per student costs \( c \). Suppose that both cost components are subsidized by the government at rate \( s \); then subsidy-inclusive per student costs of university \( i \) are given by \( (1-s)\left(c+\gamma q_i^2\right)\).

***Proposition 5.***
An increase in proportional cost subsidy brings the following changes:
\[
\frac{\partial q_i}{\partial s_i} > 0 \quad (i = 1, 2), \quad \frac{\partial \Delta}{\partial s} > 0, \quad \frac{\partial x_2}{\partial s} = -\frac{\partial x_1}{\partial s}, \quad \frac{\partial p_1}{\partial s} > 0, \quad \frac{\partial p_2}{\partial s} > 0, \quad \frac{\partial p_2}{\partial s} > 0,
\]
iff
\[
\frac{(5b-a)^2 + 24(b-a)^2}{64\gamma(1-s)^2} - \frac{\gamma b^2}{9} > 0 \quad \text{iff} \quad \frac{(5a-b)^2 + 24(b-a)^2}{64\gamma(1-s)^2} - \frac{\gamma b^2}{9} > 0.
\]
\begin{align*}
\frac{\partial (p_1 - p_2)}{\partial s_1} > 0, \quad \frac{\partial R_2}{\partial s_2} > 0, \quad \frac{\partial R_1}{\partial s_1} \geq 0 \quad \text{iff} \quad \frac{9(b - a)}{8\gamma (1-s)} \geq \beta, \\
\frac{\partial TS}{\partial s_i} < 0.
\end{align*}

Proof: see Appendix.

As it was shown in section 4.2, the net revenue function of the university at the quality competition stage could be represented as \((p_j - C_j) x_j = \Delta (x_j)\); that is, it depends on the quality differential and demand only. Moreover, demand, in its turn, due to (1) depends on the price differential, which is independent from the per-student fixed cost component since this component is assumed to be the same for both universities. As a result, the fact that this component is subsidized doesn’t change the net revenue function and has no implications for the quality levels. The results for the quality choice stay the same as in the case of quality investment subsidy.

Since the demands depend on the price differential, which is independent from the per-student fixed cost component, enrollment is not affected by this additional subsidy as well, and changes in the same way as under the quality investment subsidy.

However, the per-student fixed cost component, according to (2), has direct impact on tuition fee. It means that in addition to the positive strategic tuition fee effect observed under the quality investment subsidy there will be a negative effect that is due to reduced fixed per-student cost. The overall effect becomes ambiguous. The fixed-cost saving effect will be small if this component is small, thus in this an increase in tuition fee could be expected. But if the fixed-cost component contributes a lot to the total per student expenditures, then the cost-saving effect might dominate the positive strategic effect and tuition fee might go down.

\section{5.4. Tuition subsidy}

Instead of subsidizing higher education institutions, the government might subsidize students by providing tuition fee subsidy.

If the government subsidizes tuition at rate \(\tilde{s}\) then student pays \(p\) while university receives \(\tilde{p}^U\), where \(p = (1-\tilde{s})\tilde{p}^U\). Thus, the profit maximization problem of the university can be restated as

\[
\max_{p, \tilde{p}^U} \left( \frac{p_i}{1-\tilde{s}} - C_i \right) x_i(p_i, \tilde{p}^U).
\]

Multiplying by \((1-\tilde{s})\), this problem could be rewritten as

\[
\max_{p, \tilde{p}^U} \left( p_i - (1-\tilde{s})C_i \right) x_i(p_i, \tilde{p}^U).
\]

This brings the same objective function as the one from section 5.3. It means that this policy will result in the same tuition fees paid by the students, the same quality choices and students allocation as under the proportional cost subsidy considered in 5.3. Thus all the results of proposition 5 except the one that deals with universities revenue are valid for tuition subsidy as well.

Although, according to proposition 5 the changes in tuition fees paid by the students are ambiguous, the net-of-subsidy tuition fees set by universities will increase:
The net revenue of university \( i \) is:

\[
\Delta R_i = \Delta (x_i) \cdot (p_i - C_i).
\]

As it was shown in proposition 5, \( \Delta (x_i) \) increases in \( \bar{\sigma} \). Since \( 1/(1 - \bar{\sigma}) \) is also increasing in \( \bar{\sigma} \), the net revenue of the low-quality university increases in tuition subsidy. The net revenue for the high-quality university will increase as well since

\[
\frac{\partial}{\partial \bar{\sigma}} \left( \Delta (x_i) \right) > 0.
\]

5.5. Free of charge education quota at high quality university

Assume that the government provides a possibility to get education free of charge for the best students that attend a high quality university. Denote the size of this quota by \( \bar{x} \). Note that this quota under given prices and quality levels doesn’t distort the allocation of students between the two universities if quota is small enough so that \( \bar{x} < b - \bar{\theta} \). Then students with \( \bar{\theta} \in [\bar{\theta}, b] \) will choose university 1 and those with ability \( \bar{\theta} \in [b - \bar{x}, b] \) will attend it for free, while others will pay full tuition \( p_1 \) and students with \( \bar{\theta} \in [a, \bar{\theta}] \) will choose university 2.

At the price competition stage, the net revenue of university 1 becomes different as it can set tuition fee for the residual demand only and should provide its services to some students at the compensation \( s \) (subsidy) set by the government. The net revenue of the high-quality university takes the form

\[
R_{1} = \left( p_1 - C_1 \right) \left( x_i \left( p_1, p_2 \right) - \bar{x} \right) + \left( s - C_1 \right) \bar{x},
\]

while the net revenue maximization problem of university 2 stays the same.

The first order conditions result in the following system for the equilibrium tuition fees:

\[
\begin{align*}
\dot{p}_1 &= \left( b - \bar{\theta} - \bar{x} \right) \Delta + C_1; \\
\dot{p}_2 &= \left( \bar{\theta} - a \right) \Delta + C_2.
\end{align*}
\]

(12)

Plugging these prices into (1) and rearranging, the following expression is derived

\[
\dot{\bar{\theta}} = \frac{b + a - \bar{x} + \gamma (q_1 + q_2)}{3} \beta (b - a) - \frac{6 (q_1 - q_2)}{6 (q_1 - q_2)}.
\]

(13)

Now let’s proceed to the analysis of the quality competition stage. Plugging tuition fees (12) into net revenue functions, we get

\[
R_1 = \left( p_1 - C_1 \right) \left( x_i \left( p_1, p_2 \right) - \bar{x} \right) + \left( s - C_1 \right) \bar{x} = \left( \bar{\theta} - \bar{x} \right) \Delta + \left( b - \bar{\theta} - \gamma q_i \bar{x} \right)
\]

and

\[
R_2 = \left( p_2 - C_2 \right) \left( \bar{x} \right) = \left( \bar{\theta} - a \right) \Delta,
\]

which leads to the following optimization problems for the quality competition stage:

\[
\begin{align*}
\max_{q_1 \geq 0} \left( \bar{\theta} - \bar{x} \right) \Delta + \left( b - c - \gamma q_1 \right) \bar{x} \quad \text{s.t.} \quad \bar{\theta} = \frac{b + a - \bar{x} + \gamma (q_1 + q_2) - \beta (b - a)}{6 \Delta},
\end{align*}
\]

and

\[
\begin{align*}
\max_{q_2 \geq 0} \left( \bar{\theta} - \bar{a} \right) \Delta \quad \text{s.t.} \quad \bar{\theta} = \frac{b + a - \bar{x} + \gamma (q_1 + q_2) - \beta (b - a)}{6 \Delta}.
\end{align*}
\]
Taking the first order conditions, the following system is obtained:

$$
\begin{align*}
\left[ (b - \hat{\theta} - \bar{x})(b - \hat{\theta} - \bar{x} - 2\Delta\gamma / 3) - 2\gamma q_x \bar{x} = 0; \\
-(\hat{\theta} - a) + 2\Delta \left( \frac{\gamma}{3} - \frac{\beta(b - a)}{6\Delta^2} \right) = 0.
\end{align*}
$$

(S14)

Solving this system for the case of zero peer effect intensity ($\beta = 0$), we get

$$
\begin{align*}
\gamma q_x &= 3\gamma q_x - 2a - \bar{x} + b, \\
q_x &= \frac{\gamma(2b + 14a + 25\bar{x}) - \sqrt{D}}{32\gamma^2},
\end{align*}
$$

(S15) \quad (S16)

where $D = \gamma^2(2b + 14a + 25\bar{x})^2 - 32\gamma^2[(b + a - \bar{x})(-b + 5a + \bar{x}) - 18\bar{x}(b - 2a - \bar{x})]$, satisfies the required constraint $\bar{x} < b - \hat{\theta}$.

Now let’s analyze how the introduction of tuition-free quota at the high-quality university will affect the strategic competition.

**Proposition 6.**

In the absence of the peer effect, the introduction of the differentially small tuition-free quota at high-quality university brings the following results:

$$
\begin{align*}
\frac{\partial q_x}{\partial x} |_{x=0} &< 0, \quad \frac{\partial p_i}{\partial x} |_{x=0} < 0 \quad (i=1, 2), \\
\frac{\partial \Delta}{\partial x} |_{x=0} &< 0, \quad \frac{\partial x_1}{\partial x} |_{x=0} > 0, \quad \frac{\partial (x_i - \bar{x})}{\partial x} |_{x=0} < 0, \\
\frac{\partial x_2}{\partial x} |_{x=0} &< 0, \quad \frac{\partial (p_1 - p_2)}{\partial x} |_{x=0} < 0, \quad \frac{\partial R}{\partial x} |_{x=0} < 0 \quad (i=1, 2), \quad \frac{\partial TS}{\partial x} |_{x=0} > 0.
\end{align*}
$$

Proof: see Appendix.

The state-financed quota provided to the best students in the high-quality university reduces the potential pool of applicants for the paid enrollment and the remaining applicants have lower quality preferences. As a result, the universities will find it profitable to reduce quality investment, moreover, university 1 reduces its quality investment more than university 2 due to significant reduction in demand for paid enrollment so that the quality differential goes down. Although the paid enrollment of university 1 goes down, the overall enrollment increases due to the state-financed quota, that is, the crowding out of paid enrollment is only partial. Reduced costs that result from the investment cost economy explain the reduction in tuition fees. As this cost reduction is higher for university 1, the fall in tuition is also higher and the tuition differential is reduced.

Reduced enrollment and reduced tuition fee result in a lower net revenue for the low-quality university. The net revenue of the high quality university also goes down. Although it attracts more students, it gets lower revenue due to reduced tuition fees and the per student subsidy paid by the government to compensate the absence of tuition for high-talented students might be even lower. Thus, the net revenue from each student goes down and this reduction is not compensated by the increased enrollment.

In the absence of peer effect, the students’ allocation between the two universities is socially efficient. Introduction of the state-financed quota
distorts this allocation and enrollment changes in favor of the high-quality university that reduces social welfare. Simultaneously, a reduction in quality differential is observed that reduces excessive investment in product differentiation by the high quality university, which is beneficial for the society. If the introduced quota is small then the quality effect is dominant and the total surplus increases as it is indicated in proposition (6) but the result might change for larger quota (Fig. 1).

5.6. Minimum quality standard
In most of countries higher education is subject to some form of quality regulation. This regulation usually takes the form of licensing. In order to get the license, an education institution should satisfy some minimum quality requirement. Let us investigate how this minimum quality regulation will affect the strategic competition between the two universities.

Suppose that the government introduces some minimum quality requirement $\bar{q}$ and this requirement is binding for low quality university, so that $q_2 = \bar{q} \geq \frac{5a-b}{8\gamma} - \frac{\beta}{3}$. Assume also that the minimum quality requirement does not exceed the efficient level for the low quality university, that is, $\bar{q} \leq \frac{b+3a}{8\gamma}$.

The price competition stage does not change, so we proceed directly to the quality competition. Since the minimum quality requirement is assumed to be binding for the low-quality university, then $q_2 = \bar{q}$ and only the quality level for high-quality university should be derived. The optimal quality is derived from the following problem

$$
\max_{q_1} \Delta \left( b - \hat{\theta} \right)^2 \quad \text{s.t.} \quad \hat{\theta} = \frac{\gamma \left( q_1 + \bar{q} \right)}{3} + \frac{\left( b + a \right)}{3} - \frac{\beta \left( b-a \right)}{6\Delta}.
$$

This problem was solved in section 4.2 (technical details could be found in Appendix). Thus, the optimal quality level of university 1 is given by

$$
q_1 = \left( 4\gamma \bar{q} + 2b - a + \sqrt{D} \right) / 6\gamma,
$$

where $D = (2b - a - 2\gamma \bar{q})^2 - 6\gamma \beta (b-a) \geq 0$ if peer effect is small enough.

Now let’s evaluate how the policy of minimum quality standard affects the equilibrium. The results of this analysis are summarized in Proposition 7.
Proposition 7.

An introduction of some minimum quality requirement (starting from the equilibrium quality level of the low-quality university) brings the following results:

\[
\begin{align*}
\frac{\partial q}{\partial q} &> 0 \quad (i = 1, 2), \\
\frac{\partial \Delta}{\partial q} &< 0, \\
\frac{\partial x_1}{\partial q} &< 0, \\
\frac{\partial x_2}{\partial q} &> 0,
\end{align*}
\]

\[
\begin{align*}
\frac{\partial p_2}{\partial q} &> 0, \\
\frac{\partial p_1}{\partial q} &< 0 \quad \text{if } a \leq 3b/7
\end{align*}
\]

\[
\begin{align*}
\frac{\partial (p_1 - p_2)}{\partial q} &< 0, \\
\frac{\partial R_1}{\partial q} &< 0, \\
\frac{\partial R_2}{\partial q} &> 0 \quad \text{and } \frac{\partial TS}{\partial q} > 0.
\end{align*}
\]

Proof: see Appendix

The results indicated in proposition (7) could be explained intuitively. The quality of university 2 will go up with an increase in the minimum quality standard as it is binding for the low-quality university. This reduces quality differential and intensifies price competition. To relax this competition, the high-quality university increases its quality investment but less than the low-quality university, and quality differential goes down.

Due to significant quality improvement, university 2 attracts more students while the enrollment of university 1 decreases. As a result, there are three changes that affect the equilibrium prices: increased quality investment drives up the per student cost that increases tuition fees; reallocation of demand in favor of university 2 reduces the price of university 1 and increases the price of university 2; and reduced quality differential makes the price competition more intense that reduces the equilibrium prices of both universities. As the change in quality for university 2 exceeds the one for university 1, then it should be expected that the effect of increased marginal cost would be dominating for university 2. For university 1, the relative strength of these effects depends on the parameters of the model so that the overall change is ambiguous. Increased tuition fee, together with increased enrollment, explains the increase in the net revenue of the low-quality university but the net revenue of the high-quality one goes down due to reduced demand and increased teaching costs associated with the increased quality.

There are two effects for the social welfare. The binding quality standard solves (at least partially) the problem of underinvestment by the low quality university but simultaneously it aggravates the overinvestment problem for the high-quality university. It also improves the student allocation since the unregulated case in the presence of peer effect involves too high enrollment to the high-quality university. The overall change in social welfare is positive if the minimum quality level is close to the equilibrium quality level of the low-quality university.

It should be noted that further increase in quality standard might reduce social welfare. With rather high (close to the efficient level for the low-
quality university) minimum quality requirement, the quality level of the high-quality university might be too high in comparison with the socially efficient one, which is too expensive for the economy and might reduce the social welfare. This possibility is illustrated by Fig. 2, where the small levels of minimum quality requirement improve the social welfare. When quality requirement becomes close to the efficient level, which is equal to 0.5 in this example, it could be observed that further increase in minimum quality requirement reduces the social welfare.

5.7. Comparison

The results of policy analysis are summarized in table 1. Comparing the implications of the student financial aid policy represented by tuition subsidy with different kinds of cost subsidies, it can be seen that these policies have similar impacts on all the variables excluding tuition fees and net institutional revenues. In particular, those policies result in then increase in both tuition and education qualities differential and reallocate enrollment in favor of the low-quality university.

Taking into account that, in the absence of regulation, the quality differentiation was excessive and too many students were attracted by the high quality university, these policies have two effects for the social welfare: the positive impact due to better enrollment allocation and the negative impact due to excessive product differentiation, but the later effect dominates and the total surplus goes down. However, these policies may have different impact on the composition of the total surplus; for example, they may have different impact on the net revenue of the high-quality university if the peer effect is big enough.

It should be noted that not all funding policies are alike. If, instead of the universal subsidy rates that are identical for both universities, a policy that offers free-of-charge education to the most able students is introduced, then this policy will bring quite different results. It reduces both quality and tuition fee differential and brings some reallocation of students in favor of the high-quality university. This improves both the welfare of students and social welfare. This opposite impact takes place due to the asymmetric nature of this policy. Contrary to the subsidies considered above, this policy has direct impact on the high-quality university only since the low-quality university doesn’t get the possibility to admit students at tuition-free basis. Since best students are admitted for free, the universities compete for the reduced
### Table 1

Effects of government policy on the equilibrium

<table>
<thead>
<tr>
<th>Policy</th>
<th>Quality levels</th>
<th>Quality differential</th>
<th>Enrollment</th>
<th>Tuition fees set by universities</th>
<th>Tuition differential</th>
<th>Net revenue = Research fund</th>
<th>Social welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Better peer effect ($\beta \uparrow$)</td>
<td>$q_1$</td>
<td>$q_2$</td>
<td>$\Delta$</td>
<td>$x_1$</td>
<td>$x_2$</td>
<td>$p_1$</td>
<td>$p_2$ $p_1 - p_2$</td>
</tr>
<tr>
<td>Stricter minimum admission requirements</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>Quality investment subsidy</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$ ($\beta$ — low)</td>
</tr>
<tr>
<td>Proportional cost subsidy</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$ ($\beta$ — high)</td>
</tr>
<tr>
<td>Tuition subsidy</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$ ($\beta$ — low)</td>
</tr>
<tr>
<td>Introduction of tuition-free education for best students in the absence of peer effect ($\beta = 0$)</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>Introduction of minimum quality standard starting from $q = q_2$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$ $+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$ $+$</td>
</tr>
</tbody>
</table>

Social welfare

#### Notes:
- $+$ positive effect
- $-$ negative effect
- $+$ (low) or $+$ (high) indicates the effect for low or high values of the corresponding parameter.
pool of students, which is less diverse in terms of their abilities and has lower quality preferences. This reduces incentive for quality investment, which, in its turn, reduces excessive product differentiation and finally results in the higher total surplus.

Thus, the total surplus under the introduction of tuition-free quota goes up and so, from the efficiency point of view, this policy outperforms both universal tuition and cost subsidies. But it should be taken into account that (given that the per-student grant provided by the government doesn’t exceed the market tuition fee) this policy results in reduction of net revenues of both universities, which reduces the research funds. As the proposed model is static, the importance of universities’ investment in research and development that are crucial for the long-run economic growth could be underestimated. Thus, if the society cares about the universities’ research funds, then some additional research grant might be required to compensate for the reduced income from teaching.

Finally, the implications of direct regulation in the form of minimum admission requirements, quality standard and some curriculum requirements that may affect the peer effect intensity should also be taken into account.

Since in the absence of regulation the education quality chosen by the low quality university is below the efficient level, a slight increase in quality that comes from the binding quality standard is beneficial for the economy. This happens because this policy reduces the excessive quality differentiation and also solves the problem of excessive enrollment to the high-quality university that appears due to the peer effect. Thus, it can be concluded that some quality regulation is socially desirable. But this analysis is valid for a small increase in the quality standard. If the quality standard becomes too high then further increase in quality standard might reduce social welfare as it was illustrated by Fig. 2.

Although the stricter minimum admission requirement has the same impact on quality as the minimum quality standard, that is, it reduces excessive product differentiation; the overall outcome is quite different. This is explained by the reduction of potential pool of students that decreases enrollment and tuition for both universities and, as a result, brings the fall in universities’ net revenue from education and in the social welfare. Thus, we can conclude that the tight government admission regulation is undesirable. In the proposed modeling approach, zero drop-out rate was assumed. It was motivated by the effective level of minimum admission regulation that prevents those students that are unable to cope with the program from entering the university. Any regulation in excess of this minimum level will reduce social welfare.

It was shown that the peer effect has positive impact on students and social welfare. Although the government cannot influence the peer effect intensity directly, it can stimulate universities to encourage students’ group work and other forms of students’ interactions that facilitate
the peer effect. For example, it can be implemented via some curriculum requirements.

Conclusions

In this paper, a strategic competition between two universities has been modeled using a vertical product differentiation model, where at the first stage of the game the two universities compete by choosing education quality investment and at the second stage they compete by setting tuition fees. The model incorporates a positive external effect that is specific for the education sector and is known as the peer effect.

It was demonstrated that, in the absence of regulation, the product differentiation is excessive: one university provides the quality of education which exceeds the socially efficient one, while the other university provides the quality which is below the socially efficient one so that the quality differential is too high in comparison with the efficient level. This result is quite standard for the models of vertical differentiation. It was also shown that in the presence of the peer effect the students’ allocation is inefficient as too many students choose the high-quality university. This result is absent in standard vertical product differentiation model and arises here due to additional benefit that comes from the peer effect, which is stronger in the high-quality university.

The positive externality that comes from the peer effect makes education more attractive. As a result, with higher peer effect intensity universities economize on costs by reducing investment in education quality. Since the high-quality university attracts better students, the peer effect is more pronounced in this university and increased peer effect intensity reallocates enrollment from the low-quality to the high-quality university. This is beneficial for the high-quality university and its net revenue goes up while the net revenue of the low-quality one goes down. It was also demonstrated that increased peer effect intensity improves both students’ and social welfare. The government may affect the intensity of the peer group effect by increasing the role of group work in educational standards.

The policy analysis suggests that both demand-side and supply-side policies have similar impact on the quality investment, enrollment and social welfare if these polices are uniform (the subsidy rates are applied for all students or all universities). In particular, tuition fee subsidy, universities’ cost subsidy and quality investment subsidy were considered. It was demonstrated that all these policies result in an increase in the quality investment with additional product differentiation (an increase in quality differential), raising tuition fee differential, bringing some reallocation of enrollment from the high-quality to the low-quality university, and reducing social welfare. But a combination of demand-side and supply-side policies that gives the possibility for the most talented student to get education for free in the high-quality university and provides per student grant to the high-quality university to finance the education of those students results in quite different
outcome for quality investment, enrollment, and social welfare. Since this policy reduces the demand for paid enrollment, it makes the quality investment less profitable and reduces quality investment and excessive product differentiation. The cost economy that results from the reduced spending on the quality investment leads to lower tuition fees that increase students’ welfare. Finally, the overall social welfare also goes up.

The analysis demonstrates that the tight minimum admission requirements are not desirable while a slight increase in minimum quality standard above the equilibrium quality level of the low-quality university has positive impact on both students’ and social welfare. At the same time, if the minimum requirement level becomes high (close to the socially efficient level of the low-quality university), then further increase in the quality standard may reduce social welfare.

The analysis was undertaken for the case of interior solution that requires some restrictions on students’ heterogeneity and the peer effect intensity. Further analysis is required to deal with the corner solutions, where the parameters are such that only one out of the two universities undertakes the quality investment in the equilibrium.

It should be noted that the results were derived for the model that accounts for direct peer effect only and so does not take into consideration the indirect peer effect that works through the change in the agents behavior (both students and professors) and might, in addition to the average student ability, depend on other group characteristics, for example, groups heterogeneity.

Some other limitations are due to specific assumptions that deal with cost specification and distribution of students’ abilities. The analysis relies on uniform distribution of students’ abilities and quadratic cost function for quality investment. These assumptions are quite standard for the models of vertically differentiated products but some alternative assumptions might be considered as well.

The analysis could be extended to other forms of government policy, when the government funding or students’ support is targeted (for example, subsidies could be provided to the university with some targeted level of quality investment) or allocated on a competitive basis. To model the universities competition for the state funding, the game should include additional stage, where an exogenously fixed government budget is allocated by the planner who maximizes social welfare. This stage might take place after the quality investment decisions and then the quality competition will play dual role: on the one hand, the quality investment will be used as an instrument of government funding competition and, on the other hand, it will increase education premium and increase demand for the given higher education institution.

In addition to the quality and admission regulation considered in the paper some other forms of government policies, for example, tuition regulation might be considered.
APPENDIX

Second order conditions for the quality choice problem

The second order condition (SOC) for the quality choice problem of university 1 is
\[
\beta - \gamma + \Delta + \frac{\beta (b-a)}{6 \Delta^2} \leq 0. \tag{A1}
\]
Taking into account the FOC, it can be restated as:
\[
-\theta \leq \gamma \Delta. \tag{4}
\]
Plugging expression (4) for \(\hat{\theta}\) and using FOC it takes the form
\[
(\beta - \gamma - \theta + \Delta) + \frac{\beta (b-a)}{6 \Delta^2} \leq 0. \tag{A1}
\]
Extremum for university 1 is given by the solution of the following equation:
\[
3q_1^2 - (4q_2 + 2b-a)q_1 + \gamma q_2^2 + (2b-a)q_2 + \beta (b-a) / 2 = 0.
\]
If the peer effect is small\(^5\) this equation gives two roots
\[
q_{11} = \left(4q_2 + 2b-a\right) / 6\gamma
\]
and
\[
q_{12} = \left(4q_2 + 2b-a - \sqrt{D}\right) / 6\gamma,
\]
where
\[
D = (2\gamma q_2 - (b-a))^2 - 6\gamma \beta (b-a),
\]
but only the first root satisfies (A1).

The SOC for university 2 is:
\[
(\beta - \gamma + \Delta + \frac{\beta (b-a)}{6 \Delta^2}) \leq 0. \tag{A1}
\]
Taking into account the FOC and plugging expression (4) for \(\hat{\theta}\) it can be restated as
\[
\Delta (\beta - \gamma + \Delta + \frac{\beta (b-a)}{6 \Delta^2}) \leq 0. \tag{A1}
\]
This inequality is always satisfied for any \(\Delta > 0\) and \(b \geq 2a\).

Proof of Lemma 1.

The FOCs for total surplus maximization problem take the form:
\[
TS_0' = (q_2^* - q_1^*)\theta - \gamma (q_2^*)^2 - (q_1^*)^2 \geq 0, \theta^* = (a, b); \tag{A2}
\]
\[
TS_0' = 0.5 (b - \theta^*)^2 - 2\gamma q_1^* (b - \theta^*) \leq 0, q_1^* = 0; \tag{A3}
\]
\[
TS_0' = 0.5 (\theta^* - a)^2 - 2\gamma q_2^* (\theta^* - a) \leq 0, q_2^* = 0; \tag{A4}
\]
If both universities are active then \(a < \theta^* < b\) and condition (A2) implies
\[
(q_2^* - q_1^*) (\theta^* - \gamma (q_1^* + q_2^*)) = 0. \tag{A2'}
\]
As \((\theta^* - a) > 0\) and \((b - \theta^*) > 0\) then (A3) and (A4) could be restated as:
\[
0.5 (b + \theta^*) - 2\gamma q_1^* \leq 0, q_1^* = 0; \tag{A3'}
\]
\[
0.5 (\theta^* + a) - 2\gamma q_2^* \leq 0, q_2^* = 0. \tag{A4'}
\]
Note that \(q_1^* > 0\) and \(q_2^* > 0\) as otherwise either (A3') or (A4') is violated. For positive quality investment (A3') and (A4') imply \(q_1^* = (b + \theta^*) / 4\gamma\) and \(q_2^* = (\theta^* + a) / 4\gamma < (b + \theta^*) / 4\gamma = q_1^*\). Since \(q_1^* - q_2^* > 0\)

\(^5\) This assumption means that \(D = (2\gamma q_2 - (b-a))^2 - 6\gamma \beta (b-a) \geq 0\), which is the case for \(\beta \leq (2\gamma q_2 - (b-a))^2 / 6\gamma (b-a)\).
then (A2') implies \( \theta' = \gamma (q_1' + q_2') \). Plugging \( q_1' \) and \( q_2' \) and solving the equation, we get \( \theta' = (b + a) / 2 \), which implies \( b - \theta' = (b - a) / 2 \). Plugging \( \theta' = (b + a) / 2 \) into \( q_1' \) and \( q_2' \), the efficient quality levels \( q_1' = (3b + a) / 8\gamma \) and \( q_2' = (b + 3a) / 8\gamma \) are derived. This allocation results in the following value of social welfare:

\[
TS_{\theta'-(b+a)/2} = \frac{b-a}{2} \left( 2(\alpha - c) + \beta (b + a) + \left[ (b + 3a)^2 + (3b + a)^2 \right] \right) / 64\gamma.
\]

Now, suppose that only one university is active. Then enrollment and quality investment for the other university will be zero and the quality investment for the active university 1 could be found from (A3):

\( q_1 = \left( b + \theta' \right) / 4\gamma = (b + a) / 4\gamma \) as \( \theta' = a \). The social welfare in this case equals

\[
TS_{\theta'=(b+a)/2} = \frac{b-a}{64\gamma} (b-a)^2 > 0.
\]

This proves that efficiency requires \( a < \theta' < b \).

**Proof of proposition 1**

Since \( \Delta' = q_1' - q_2' = \frac{b-a}{4\gamma} \) then \( \Delta = \frac{3(b-a)}{4\gamma} > \frac{b-a}{4\gamma} = \Delta' \).

Under small peer effect intensity \( \beta < -\frac{5}{8} \) we get:

\[
q_2' - q_1' = \frac{a-b}{4\gamma} < 0 \quad \text{and} \quad q_1' - q_1' = \frac{b-a}{4\gamma} - \frac{\beta}{b-a} - \frac{5a-b}{8\gamma} = \frac{3b-a}{8\gamma} > 0.
\]

Due to (5) \( b - \hat{\theta} = \frac{b-a}{2} + \frac{4\beta\gamma}{9} < \frac{b-a}{2} = b - \theta' \) if \( \beta > 0 \). This implies that enrollment to the low quality university is below the efficient level:

\[
\hat{\theta} - a = \frac{b-a}{2} - \frac{4\beta\gamma}{9} < \frac{b-a}{2} = \theta' - a.
\]

**Proof of proposition 2**

By differentiating (7) it could be found that \( \frac{\partial q_1}{\partial \beta} = \frac{\partial q_2}{\partial \beta} = -\frac{1}{3} < 0 \), which implies \( \frac{\partial \Delta}{\partial \beta} = \frac{\partial q_1}{\partial \beta} - \frac{\partial q_2}{\partial \beta} = 0 \). Differentiating (5), we get

\[
\frac{\partial \hat{\theta}}{\partial \beta} = \frac{4\gamma}{9} < 0 \quad \text{and} \quad \frac{\partial x_2}{\partial \beta} = \frac{\partial (\hat{\theta} - a)}{\partial \beta} = \frac{\partial \hat{\theta}}{\partial \beta} < 0, \quad \frac{\partial x_1}{\partial \beta} = \frac{\partial (b - \hat{\theta})}{\partial \beta} = \frac{\partial \hat{\theta}}{\partial \beta} > 0.
\]

By differentiating (8) and (9) and taking into account the upper bound for peer effect \( \beta < 3(5a-b) / 8\gamma \) it could be found that tuition fees will go down:
partial derivatives of the price functions with respect to \( \beta \):

\[
\frac{\partial p_1}{\partial \beta} = \frac{b + 3a + 2\gamma \beta}{12} < \frac{b + 3a + 5a - b}{12} = \frac{a - b}{6} < 0,
\]

\[
\frac{\partial p_2}{\partial \beta} = \frac{3b + a + 2\gamma \beta}{12} < \frac{3b + a + 5a - b}{12} = \frac{a - b}{3} < 0,
\]

and tuition fee differential will increase

\[
\frac{\partial p_1}{\partial \beta} - \frac{\partial p_2}{\partial \beta} = \frac{b - a}{6} > 0.
\]

As the net revenue of university \( i \) equals

\[
\frac{\partial R_i}{\partial \beta} = 2\Delta \frac{\partial x_i}{\partial \beta} > 0,
\]

that is, it will decrease for the low-quality university and increase for the high-quality one.

Differentiating TS with respect to \( \beta \) we get:

\[
\frac{\partial TS}{\partial \beta} = (b - a) \left( \frac{a + b}{2} + \frac{2\beta \gamma}{27} \right) > 0.
\]

**Proof of proposition 3**

Differentiating (7) it could be found that

\[
\frac{\partial q_1}{\partial a} = -\frac{1}{8\gamma} < 0 \quad \text{and} \quad \frac{\partial q_2}{\partial a} = \frac{5}{8\gamma} > 0,
\]

which implies a reduction in the quality differential

\[
\frac{\partial \Delta}{\partial a} = \frac{\partial q_1}{\partial a} - \frac{\partial q_2}{\partial a} = -\frac{3}{4\gamma} < 0.
\]

It follows from (5) that \( \partial \hat{\theta} / \partial a = 0.5 > 0 \), which results in equal reduction of enrollment \( \partial x_i / \partial a = \partial (b - \hat{\theta}) / \partial a = -0.5 < 0 \) and \( \partial x_2 / \partial a = \partial \left( \hat{\theta} - a \right) / \partial a = -0.5 < 0 \).

Differentiating (8) and (9) and evaluating the signs (taking into account that \( b > 2a \)), we get:

\[
\frac{\partial p_1}{\partial a} = -\frac{25(b - a) + 4b}{32\gamma} < 0 \quad \text{and} \quad \frac{\partial p_2}{\partial a} = \frac{49a - 29b}{32\gamma} - \frac{\beta}{12} < -\frac{9a}{32\gamma} - \frac{\beta}{12} < 0,
\]

but the tuition fee differential will go down:

\[
\frac{\partial p_1}{\partial a} - \frac{\partial p_2}{\partial a} = \frac{3a}{4\gamma} - \frac{\beta}{6} < 0.
\]

The net revenue will decrease due to reduced enrollment and quality differential:

\[
\frac{\partial R_i}{\partial a} = \frac{\partial (\Delta x_i^2)}{\partial a} = \frac{\partial \Delta}{\partial a} x_i^2 + 2x_i \Delta \frac{\partial x_i}{\partial a} = \frac{3}{4\gamma} x_i^2 - x_i \Delta < 0.
\]

The net benefit from education for \( \theta = a \) must be positive:

\[
u(0 = a) = \alpha + \beta q_2 + q_2 a - p_2 > 0.
\]

Plugging equilibrium values for \( \hat{\theta} \), \( q_2 \) and \( p_2 \) given by (5), (7) and (9) and rearranging, we get

\[
u(0 = a) = \alpha - \epsilon - \frac{\beta^2 \gamma}{3} + \frac{\beta(a + b)(5b - 9a)}{64\gamma} > 0.
\]

Differentiating total surplus with respect to \( a \) and using the above stated inequality, it could be found that negative enrollment effect will dominate and the total surplus will go down:
Proof of proposition 4

Differentiating (7), we get
\[
\frac{\partial q_1}{\partial s} = \frac{b - 5a}{8 \gamma^2 (1-s)^2} > 0 \quad \text{and} \quad \frac{\partial q_2}{\partial s} = \frac{5a - b}{8 \gamma^2 (1-s)^2} > 0,
\]
which results in the increased quality differential
\[
\frac{\partial \Delta}{\partial s} = \frac{3(b - a)}{4 \gamma^2 (1-s)^2} > 0.
\]

From (5) \( \frac{\partial \hat{\theta}}{\partial s} = \frac{4 \beta}{9} > 0 \), which implies reduced enrollment to the high-quality university \( \frac{\partial x_1}{\partial s} = -\frac{\partial \hat{\theta}}{\partial s} = -\frac{4 \beta}{9} < 0 \) and increased enrollment to the low-quality one \( \frac{\partial x_2}{\partial s} = \frac{4 \beta}{9} > 0 \).

By differentiating (8) and (9) under \( \beta < 3(5a - b) / 8 \gamma \), we get that tuition fees will increase:
\[
\frac{\partial p_1}{\partial s} = \frac{(5b - a)^2}{64 \gamma^2 (1-s)^2} - \frac{\beta^2}{9} > \frac{3(b - a)}{4 \gamma^2 (1-s)^2} > 0,
\]
\[
\frac{\partial p_2}{\partial s} = \frac{(5a - b)^2}{64 \gamma^2 (1-s)^2} - \frac{\beta^2}{9} > \frac{3(b - a)^2}{8 \gamma^2 (1-s)^2} > 0,
\]
but tuition fees of the high-quality university increases more
\[
\frac{\partial p_1}{\partial s} - \frac{\partial p_2}{\partial s} = \frac{3(b^2 - a^2)}{8 \gamma^2 (1-s)^2} > 0.
\]

The net revenue from education for the low-quality university increases \( \frac{\partial R_2}{\partial s} = \frac{3(b - a)}{4 \gamma^2 (1-s)^2} x_2^2 + 2 x_2 \Delta \frac{4 \beta}{9} > 0 \) but the result for the high-quality university is ambiguous:
\[
\frac{\partial R_1}{\partial s} = x_1 \left( \frac{b - a}{\gamma (1-s)^2} \left( \frac{3(b - a)}{8 \gamma (1-s)^2} - \frac{\beta}{3} \right) \right) > 0 \quad \text{iff} \quad \frac{9(b - a)}{8 \gamma (1-s)^2} > \beta.
\]

By differentiating (TS) with respect to \( s \) and taking into account that in the interior solution \( 8 \beta \gamma (1-s) < 3(5a - b) \), we find that total surplus is diminishing in subsidy rate:
\[
\frac{\partial TS}{\partial s} = \frac{b - a}{32 \gamma (1-s)^3} \left( 2ab(6-s) - (6 + 7s)(b^2 + a^2) + \frac{64 \beta^2 \gamma^2 (1-s)^3}{27} \right) < \frac{(b - a)b(33a - 17b)}{96 \gamma (1-s)^3} < 0.
\]

Proof of proposition 5

Since the quality-independent per-student costs \( c \) do not enter equation (7) for the quality levels as well as equation (5) for the indifferent
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It was shown in proposition 4 that quality investment subsidy increases tuition fee but now in addition the fixed per-students costs are subsidized that reduces tuition fee and the overall effect is ambiguous. In the presence of proportional cost subsidy, the tuition fees given by (8) and (9) will take the following form:

\[
p_1 = c(1-s) + \frac{(5b-a)^2 + 24(b-a)^2}{64\gamma(1-s)} - \frac{\beta(b+3a)}{12} + \frac{\gamma(1-s)\beta^2}{9},
\]

\[
p_2 = c(1-s) + \frac{(5a-b)^2 + 24(b-a)^2}{64\gamma(1-s)} - \frac{\beta(3b+a)}{12} + \frac{\gamma(1-s)\beta^2}{9}.
\]

Differentiation with respect to \(s\) gives

\[
\frac{\partial p_1}{\partial s} = -c + \frac{(5b-a)^2 + 24(b-a)^2}{64\gamma(1-s)^2} - \frac{\gamma\beta^2}{9} > 0
\]

iff \(\frac{(5b-a)^2 + 24(b-a)^2}{64\gamma(1-s)^2} - \frac{\gamma\beta^2}{9} \geq c\) and

\[
\frac{\partial p_2}{\partial s} = -c + \frac{(5a-b)^2 + 24(b-a)^2}{64\gamma(1-s)^2} - \frac{\gamma\beta^2}{9} > 0
\]

iff \(\frac{(5a-b)^2 + 24(b-a)^2}{64\gamma(1-s)^2} - \frac{\gamma\beta^2}{9} \geq c\).

Whatever is the direction of tuition fee change, the fee for the high-quality university will change more than tuition fee for the low-quality university:

\[
\frac{\partial (p_1 - p_2)}{\partial s} = \frac{3(b^2 - a^2)}{8\gamma^2(1-s)^2} > 0.
\]

Since the equilibrium net revenue from education for each university \(R_i = \Delta x_i^2\) does not depend on \(c\), then the change in net revenue will also be the same as in the case of quality investment subsidy.

Differentiating TS with respect to subsidy rate, we get:

\[
\frac{\partial \text{TS}}{\partial s} = \frac{b-a}{32\gamma(1-s)^3} \times
\]

\[
\times \left(2ab(6-s) - (6 + 7s)(b^2 + a^2) + \frac{64\gamma^2(1-s)^3}{27}\right) < \frac{(b-a)(33a - 17b)}{96\gamma(1-s)^3} < 0.
\]

**Proof of proposition 6**

Differentiating (16) with respect to \(\bar{x}\) and evaluating derivative at \(\bar{x} = 0\) it could be found that university 2 will increase its quality level

\[
\frac{\partial q_2}{\partial \bar{x}} \bigg|_{\bar{x}=0} = -\frac{13b-a}{16\gamma(b-a)} < 0.
\]

Differentiating (15) it could be shown that the quality level of university 1 will also decrease:

\[
\frac{\partial q_1}{\partial \bar{x}} \bigg|_{\bar{x}=0} = -\frac{29b-17a}{16\gamma(b-a)} < 0
\]

and the quality differential
goes down \( \frac{\partial \Delta}{\partial \bar{x}} \bigg|_{x=0} = -\frac{1}{\gamma} < 0 \).

Differentiating (13) with respect to \( \bar{x} \) we get \( \frac{\partial \theta}{\partial \bar{x}} \bigg|_{x=0} = -\frac{2}{3} < 0 \). It implies that more students will choose the high-quality university \( \frac{\partial x_1}{\partial \bar{x}} \bigg|_{x=0} = \frac{2}{3} > 0 \) and less will choose the low-quality university \( \frac{\partial x_2}{\partial \bar{x}} \bigg|_{x=0} = -\frac{1}{3} < 0 \). Note that 
\[
\frac{\partial (x_1 - \bar{x})}{\partial \bar{x}} \bigg|_{x=0} = -\frac{1}{3} < 0, \text{ that is, there is partial crowding out of the paid enrollment in university 1.}
\]

According to (12), the tuition fee for university 2 goes down:
\[
\frac{\partial p_2}{\partial \bar{x}} \bigg|_{x=0} = -\frac{\Delta}{3} - \frac{x_2}{\gamma} + 2\gamma q_2 \frac{\partial q_2}{\partial \bar{x}} \bigg|_{x=0} < 0. \quad \text{From (12) } p_1 = (b - \bar{x}) + \gamma q_1^2.
\]

Differentiating with respect to \( \bar{x} \), we get
\[
\frac{\partial p_1}{\partial \bar{x}} \bigg|_{x=0} = \frac{3(b-a)}{4\gamma} - \frac{5b-a}{4} \times \frac{29b-17a}{16\gamma(b-a)} < 0.
\]

The tuition fee differential goes down:
\[
\frac{\partial (p_1 - p_2)}{\partial \bar{x}} \bigg|_{x=0} = \frac{-26ab-47b^2+21a^2}{32\gamma(b-a)} < 0.
\]

The net revenue of university 2 decreases: \( \frac{\partial R_2}{\partial \bar{x}} \bigg|_{x=0} = -\frac{1}{\gamma} (\theta-a)^2 - \frac{2}{3} (\bar{x}) \Delta < 0. \) Assuming that \( s \leq p_1 = (b - \bar{x}) + C_1 \), the net revenue of university 1 will go down:
\[
\frac{\partial R_1}{\partial \bar{x}} \bigg|_{x=0} = \left(b - \bar{x}\right)^2 \frac{\partial \Delta}{\partial \bar{x}} \bigg|_{x=0} + 2\left(b - \bar{x}\right) \Delta \left(\frac{\partial \theta}{\partial \bar{x}} \bigg|_{x=0}\right) + (s - C_1) \leq -\frac{(b-a)^2}{24\gamma} < 0.
\]

Differentiating TS we can see that the social welfare goes up:
\[
\frac{\partial TS}{\partial \bar{x}} \bigg|_{x=0} = \frac{3(b-a)(7b-3a)}{32\gamma} > 0. \quad \Box
\]

**Proof of proposition 7**

Since for university 2 the regulation is binding, then \( q_2 = \bar{q} \) and so \( q_2 \) is increasing in \( \bar{q} \). By differentiating (17) it could be found that
\[
\frac{\partial q_1}{\partial \bar{q}} \bigg|_{\bar{q}=0} = \frac{9(b-a) - 8\bar{q}\beta}{27(b-a) - 8\bar{q}\beta} > 0
\]
since \( 8\beta \gamma < 3(5a-b) < 3(5a-b) + 3(4b-8a) = 9(b-a) \) and \( 2a < b \) by the assumption. Although both universities increase quality investment, the change for the high quality university is smaller and the quality differential falls: \( \frac{\partial \Delta}{\partial \bar{q}} \bigg|_{\bar{q}=ar{q}_1} = -\frac{18(b-a)}{27(b-a) - 8\bar{q}\beta} < 0 \).
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The indifferent student ability increases

\[ \frac{\partial \hat{\theta}}{\partial q} \bigg|_{q=q_{i}} = 4\gamma \left( \frac{9(b-a) - 8\beta}{27(b-a) - 8\beta} \right) > 0, \]

which implies that the enrollment of the high-quality university goes down

\[ \frac{\partial x_1}{\partial q} \bigg|_{q=q_{i}} = -\frac{\partial \hat{\theta}}{\partial q} \bigg|_{q=q_{i}} < 0, \]

and for the low-quality one it goes up

\[ \frac{\partial x_2}{\partial q} \bigg|_{q=q_{i}} = \frac{\partial \hat{\theta}}{\partial q} \bigg|_{q=q_{i}} > 0. \]

Tuition fee of the low-quality university increases

\[ \frac{\partial p_2}{\partial q} \bigg|_{q=q_{i}} = \left( 3(5a - b) - 8\beta \right) / 12 > 0, \]

but the change in tuition fee of the high-quality university is ambiguous:

\[ \frac{\partial p_1}{\partial q} \bigg|_{q=q_{i}} = \frac{27(b-a)(7a-3b) - 32\beta(6b-3a-2\beta)}{12(27(b-a) - 8\beta)}. \]

If \( a \leq 3b / 7 \) then \( \frac{\partial p_1}{\partial q} \bigg|_{q=q_{i}} < 0 \), but it might be positive otherwise. Whatever is the change in \( p_1 \), the tuition differential goes down:

\[ \frac{\partial (p_1 - p_2)}{\partial q} \bigg|_{q=q_{i}} = -\frac{18a(b-a)}{27(b-a) - 8\beta} < 0. \]

Since \( R_i = (x_i)^2 \Delta \) and both the enrollment of university 1 and the quality differential go down, then the net revenue should also go down, but the net revenue of university 2 goes up due to increased enrollment:

\[ \frac{\partial R_2}{\partial q} \bigg|_{q=q_{i}} = \frac{(b-a)(9(b-a) - 8\beta \gamma)^2}{18(27(b-a) - 8\beta)} > 0. \]

Finally, the change in TS is positive:

\[ \frac{\partial TS}{\partial q} \bigg|_{q=q_{i}} = \frac{b-a}{18(27(b-a) - 8\beta \gamma)}(9(b-a) - 4\beta \gamma)(9(b-a) - 8\beta \gamma) > 0. \]

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Государственное регулирование на рынке высшего образования

Аннотация. В работе предложена модель стратегической конкуренции университетов, учитывающая эффект сообучения. Равновесие в данной модели характеризует неэффективное распределение студентов между вузами, смещенное в сторону университета с более высоким качеством образования. В таком равновесии имеет место слишком большая (по сравнению с эффективным уровнем) дифференциация по уровню качества образования. Показано, что такие традиционные варианты финансирования вузов и предоставления финансовой помощи студентам, как субсидии на инвестиции в качество образования, субсидирование совокупных расходов вузов на обучение или субсидирование платы за обучение, снижают общественное благосостояние. В то же время политика предоставления бесплатных мест в вузе для наиболее талантливых абитуриентов при выплате вузу подушевой субсидии, компенсирующей расходы на их обучение, влечет за собой как выигрыш для студентов, так и рост общественного благосостояния. Также продемонстрировано, что более жесткая политика приема в вузы негативно отражается на общественном благосостоянии, а введение стандартов качества, напротив, улучшает благосостояние общества.

Ключевые слова: высшее образование, стратегическая конкуренция, эффект сообучения, благосостояние.

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