When Is Monopolistic Competition the Relevant Market Structure?*

Sergey Kokovin[†] Mathieu Parenti[‡]
Jacques-François Thisse[§] Evgeny Zhelobodko[¶]

September 14, 2010

Abstract

We consider a differentiated market involving a handful of oligopolistic firms and a wide range of monopolistically competitive firms. Under various specifications of preferences, we show that the presence of small non-strategic firms is sufficient for the strategic interactions among oligopolistic firms to be diluted when the monopolistically competitive firms can freely enter and exit the market in response to oligopolistic firms' decisions. Furthermore, the oligopolistic firms behave like monopolists which face demands encapsulating monopolistically competitive firms' reactions. This has a far-reaching implication for the anti-trust policy: oligopolistic firms' equilibrium strategies are independent from the number of such firms. Our results hold for heterogeneous firms.

*We are grateful to Jim Friedman for comments and suggestions. We gratefully acknowledge the financial support from the Fonds de la Recherche Scientifique (Belgium), and the Economics Education and Research Consortium (EERC) under the grant No 08-036 (with the cooperation of the Eurasia Foundation, USAID, the World Bank, GDN, and the Government of Sweden).

[†]Novosibirsk State University and Sobolev Institute of Mathematics (Russia). Email: skokovin@math.nsc.ru

[‡]Université de Paris 1 and PSE (France). Email: mathieu.parenti@gmail.com

[§]CORE-UCLouvain (Belgium), PSE and CEPR. Email: jacques.thisse@uclouvain.be

[¶]Novosibirsk State University (Russia). Email: ezhel@ieie.nsc.ru

1 Introduction

Many industries involve a handful of big/multi-product firms, which are able to manipulate the market outcome, as well as a vast number of small/single-product firms, which are unable to exert an appreciable influence on the market; the best they can do is to adjust optimally to big firms' actions. Evidence is provided by laws passed in many countries, which all aim to protect small businesses against large competitors. Yet, to the best of our knowledge, such a mixed market structure has been overlooked in the literature. This paper shows that neglecting the presence of a competitive fringe is unwarranted because the presence of small and non-strategic firms may deeply affect the way the market works.

Our aim is to study a mixed market structure that blends oligopolistic and monopolistically competitive firms, i.e. firms that differ in "kind." This difference is captured by assuming that the big firms are described by *atoms* because they are able to influence the market, whereas the small firms are treated as a continuum of *negligible* agents who have no impact on the market (Aumann, 1964). Furthermore, because they bear large sunk costs, big firms are driven by commitments that do not apply to small firms which display a more versatile and flexible behavior. Accordingly, the market process is described by two-stage game in which the big firms move first by choosing their prices (or outputs), whereas the small businesses react by entering or exiting the market according to the zero-profit condition.

Because the field of monopolistic competition has been very much dominated by the Dixit-Stiglitz (1977) model, we find it natural to begin with the CES utility function. Under these preferences, we show that *the size of the competitive fringe acts as a buffer that allows the big firms to behave like quasi-monopolists.* Furthermore, as long as the entry of multi-product firms does not drive the competitive fringe out of business, firms' equilibrium strategies are independent from the number of big firms. In contrast, when the competitive fringe vanishes, multi-product firms compete strategically. All of this has a far-reaching implication: *the presence of the competitive fringe washes out the strategic interactions among big firms.* In other words, monopolistic competition à la Dixit-Stiglitz emerges as the actual market structure in an environment involving different kinds of firms. The CES displaying very special features, it is critical to investigate the robustness of those results. First, we show that they hold for heterogeneous firms à la Melitz (2003), be they big or small. Second, we turn to the case of additive preferences, which have the merit of providing new impetus to empirical analysis (Houthakker, 1960). This is an important issue because the CES equilibrium price and output are independent from the market size, whereas alternative specifications of additive preferences generate either pro-competitive effects (equilibrium prices are lower in a bigger market) or anti-competitive effects (prices are higher in a bigger market), thus implying that the generalization of the above results to general additive preferences is far from being straightforward (Zhelobodko et al., 2010). Rather unexpectedly, *all the results obtained under the CES remain valid under general additive preferences.* This extension is appealing from the empirical point of view because alternative preference specifications will typically provide theoretical predictions that are sufficiently simple to be tested, sufficiently general to make sense on an empirical level, and precise enough to allow one to discriminate between different explanations of firms' behavior.

The common thread of our approach is the existence of a market statistic, the marginal utility of income, which subsumes firms' behavior. More precisely, what drives our results is the fact that the value of this statistic is determined by the entry or exit of small firms. In other words, the marginal utility of income is independent from the big firms' actions. Since the big firms correctly anticipate the second stage outcome, they accurately treat this statistic as a parameter. In doing so, they face demands that no longer depend on the other big firms' strategies, which leads them to compete non-strategically! The market outcome with muli-product firms is thus formally identical to the one that a vast number of single-product firms would generate. This result is important because *our paper provides sound theoretical foundations to the use of monopolistic competition in many empirical analysis* (Bernard et al., 2007). Our approach also has implications for the so-called "new new trade theory." For example, heterogeneity à la Melitz has been criticized for modeling firms as different "in size but not in kind" (Neary, 2010). Our analysis shows that differences in kind are immaterial under additive preferences. Indeed, despite the fundamental difference in kind stressed here between large and small firms, in equilibrium they all adopt the same pricing rule and non-strategic market behavior.

When preferences are non-additive, we show that large and small firms interact in more convoluted ways. This breaks the stabilizing impact of the competitive fringe once the big firms' behavior impacts on the pricing behavior of the small ones. Consequently, the way the "kind" of firms matters for the market outcome depends on the nature of individual preferences.

The remainder of the paper proceeds as follows. Section 2 presents the basic model. In Section 3, we prove our main results under the CES. Section 4 extends the analysis to cope with heterogeneous firms à la Melitz. In the subsequent section, we show that the results of Section 3 can be generalized to the case of general additive preferences. Section 6 illustrates the role played by this assumption by showing that our results do not necessarily hold when preferences are non-additive. Section 7 concludes.

2 The basic model

The economy involves one horizontally differentiated good and one production factor - labor. As mentioned in the introduction, our aim is to study a mixed setting in which big and small firms adopt different market behaviors: *big* (oligopolistic) firms can manipulate the market aggregates; *small* (monopolistically competitive) firms are unable to influence market aggregates, but react to big firms' behavior through entry or exit. Recall that our goal is to model big and small firms as firms different in kind. We capture this difference by assuming that the supply side of the economy involves (i) a continuum M of single-product (SP-) firms and (ii) N (MP-) multi-product firms, where firm $j = 1, .., N$ supplies a given range $[0, n_j]$ with $n_j > 0$. The purpose of this assumption is to allow each MP-firm to manipulate the market outcome. The endogenous determination of the size of firms' product ranges is beyond the scope of this paper.[1] Following the terminology used in the dominant firm model, the continuum of small firms will be referred to as the *competitive fringe*.

On the demand side, there are L consumers who share the same CES preferences given by

$$\mathcal{U} = \int_0^M x_i^\rho \mathrm{d}i + \sum_{j=1}^N \int_0^{n_j} X_{jk}^\rho \mathrm{d}k \tag{1}$$

with $0 < \rho < 1$. In (1), x_i is the quantity of the variety provided by the SP-firm i, while X_{jk} is the quantity of the variety k provided by the MP-firm j.[2] Alternatively, preferences could be described as follows (Shimomura and Thisse, 2009):

$$\mathcal{U} = \int_0^M x_i^\rho \mathrm{d}i + \sum_{j=1}^N X_j^\rho. \tag{2}$$

Under this specification, a big firm supplies a single variety that affects positively consumers' behavior, while small firms remain negligible. In what follows, we will focus on (1). It is worth stressing, however, that the both specifications lead to the same qualitative results.

Each consumer supplies inelastically one unit of labor and owns $1/L$ of each firm.[3] The labor market is perfectly competitive and labor is the numéraire. Each consumer maximizes her utility (1) subject to the budget constraint

$$\int_0^M p_i x_i \mathrm{d}i + \sum_{j=1}^N \int_0^{n_j} P_{jk} X_{jk} \mathrm{d}k \leq y \equiv 1 + \frac{1}{L}\sum_{j=1}^N \Pi_j + \frac{1}{L}\int_0^M \pi_i \mathrm{d}i \tag{3}$$

where p_i denotes the price of the variety produced by the SP-firm i and P_{jk} the price of variety k produced by the MP-firm j, while Π_j stands for the profit earned by the MP-firm j and π_i for the profit made by the SP-firm i. Observe that the income (or expenditure) y is endogenous and determined at the market equilibrium through the redistribution of profits.

Let $\sigma = 1/(1-\rho)$ be the elasticity of substitution across varieties. First-order conditions for utility maximization with respect to each variety $i \in [0, M]$ and each variety $k \in [0, n_j]$ of firm $j = 1, ..., N$ yields the following consumer demand functions:

$$d_c(p_i) = \left(\frac{\lambda p_i}{\rho}\right)^{-\sigma} \qquad D_c(P_{jk}) = \left(\frac{\lambda P_{jk}}{\rho}\right)^{-\sigma}$$

[1] Parenti (2010) studies the determination of the product range under linear-quadratic preferences.

[2] Throughout the paper, variables associated with MP-firms are described by capital letters and those corresponding to SP-firms by lower case letters.

[3] This assumption involves no loss of generality because the structure of firms' ownership does not matter for our results.

where

$$\lambda = \frac{\rho}{Y^{1-\rho}\mathbb{P}^{\rho}}. \tag{4}$$

is the Lagrange multiplier, that is, the marginal utility of income. In (4), $Y = Ly$ is the *total income* and $\mathbb{P}$ the *total price index* that satisfies

$$\mathbb{P}^{1-\sigma} \equiv \int_0^M p_i^{1-\sigma}\mathrm{d}i + \sum_{j=1}^{N}\int_0^{n_j} P_{jk}^{1-\sigma}\mathrm{d}k \equiv \mathbb{P}_{mp}^{1-\sigma} + \mathbb{P}_{sp}^{1-\sigma} \tag{5}$$

in which $\mathbb{P}_{sp}$ stands for the price index in the SP-subsector (with the subscript sp) and $\mathbb{P}_{mp}$ for the price index in the MP-subsector (with the subscript mp). Through the redistribution of the equilibrium profits, the total income is endogenous. Plugging (4) into the demand functions, we obtain the CES *market demand* functions:

$$d(p_i) = Ld_c(p_i) = \frac{Y}{\mathbb{P}}\left(\frac{p_i}{\mathbb{P}}\right)^{-\sigma} \qquad D(P_{jk}) = LD_c(P_{jk}) = \frac{Y}{\mathbb{P}}\left(\frac{P_{jk}}{\mathbb{P}}\right)^{-\sigma}. \tag{6}$$

Let c be the marginal cost and f the fixed cost of a SP-firm. Small firm i's profits are given by

$$\pi_i = (p_i - c)d(p_i; \mathbb{P}, Y) - f.$$

Letting C_j be the marginal cost and F_j the fixed cost of the MP-firm j, its profits are:

$$\Pi_j = \int_0^{n_j}(P_{jk} - C_j)D(P_{jk}; \mathbb{P}, Y)\mathrm{d}k - F_j.$$

Hence, MP-firms are heterogeneous, whereas SP-firms are homogeneous. We will see in Section 4 how this assumption may be relaxed. The strategy of a SP-firm is its price level p_i, while the strategy of a MP-firm is a price schedule P_{jk} defined on the interval $[0, n_j]$.

The SP-firm i being negligible, its behavior does not affect the market. In contrast, since the MP-firm j is an atom of mass n_j, it influences the market when it changes its prices on a positive measure set of varieties, even though any single variety provided by this firm has a zero impact. As discussed in the introduction, the second feature that distinguishes big and small firms is the order in which they move. Our setting thus bears some resemblance with the dominant firm model in which one large firm chooses its output, anticipating the reaction of a given competitive fringe (Markham, 1951; Kydland, 1979). Besides the emphasis put on product differentiation, our setting extends this model in fundamental ways that yield new and unexpected (at least to us) results.[4]

[4]Although we believe that a sequential game does capture an essential difference between firms that differ in kind, it is worth stressing that several of our results are changed when all firms move simultaneously. Since the small firms' equilibrium strategies of the simultaneous game must solve the equilibrium conditions of the second stage, the equilibrium marginal utility of income is the same in both the sequential and simultaneous games. However, the big firms do not treat this statistic parametrically when they choose their strategies in the simultaneous game. Everything else being equal, they try to reduce the marginal utility of money, which allows them to have higher mark-ups. Consequently, big firms behave strategically, as in Shimomura and Thisse (2009) and Parenti (2010).

3 The market outcome

Because the game is sequential, we seek a subgame perfect Nash equilibrium and solve the game by backward induction.

Stage 2. Observe that $\mathbb{P}_{mp}$ encapsulates the price choices made by all MP-firms in the first stage of the game. Since each SP-firm is negligible, it accurately treats $\mathbb{P}_{mp}$ and Y as parameters. This yields the equilibrium price:

$$p_i^* \equiv p^* = \frac{\sigma}{\sigma - 1}c.$$

Therefore, for a given mass M of small firms, (5) may be rewritten as follows:

$$\mathbb{P}^{1-\sigma} = M\left(\frac{\sigma}{\sigma - 1}c\right)^{1-\sigma} + \mathbb{P}_{mp}^{1-\sigma}. \tag{7}$$

By definition of monopolistic competition, free entry and exit prevails in the SP-subsector to reflect the high turnover of small firms. As usual, the resulting zero-profit condition allows one to determine the equilibrium output of a SP-firm:

$$q_i^* \equiv q^* = \frac{f(\sigma - 1)}{c}.$$

Hence, the small firms' equilibrium price and output are independent from the big firms' choices, which implies that *the MP-firms influence the competitive fringe through the mass of SP-firms only.*

It remains to determine the equilibrium mass of SP-firms, M^*, conditional upon $\mathbb{P}_{mp}$ and Y. To this end, we consider the market clearing condition for variety $i \in [0, M]$:

$$\frac{Y}{\mathbb{P}^{1-\sigma}}\left(\frac{\sigma}{\sigma - 1}c\right)^{-\sigma} = d(p^*) = q^* = \frac{f(\sigma - 1)}{c}$$

which yields

$$\mathbb{P}^{1-\sigma} = \frac{Y}{\sigma f}\left(\frac{\sigma}{\sigma - 1}c\right)^{1-\sigma}. \tag{8}$$

Using (7) and (8), the equilibrium mass of SP-firms is given by

$$M^* = \frac{Y}{\sigma f} - \left(\frac{\sigma - 1}{c\sigma}\right)^{1-\sigma}\mathbb{P}_{mp}^{1-\sigma}.$$

When profits are not redistributed to the workers, (8) shows that the SP-subsector acts as a buffer stabilizing the total price index at a constant value through a change in the equilibrium mass M^* of SP-firms. In this case, an external shock (e.g. the entry of a big firm) that makes a subsector more (less) aggressive translates into less (more) competition in the other subsector. More generally, the equilibrium mass of SP-firms is such that the decisions made by the MP-firms affect the total price index through the income effect only.

Stage 1. The MP-firms choose their price schedules, anticipating the SP-firms' optimal responses. In particular, the MP-firms know that the mass of SP-firms will be adjusted for the total price index to remain equal to (8). Substituting (8) into $D(P_{jk})$, we obtain what we call the *adjusted demand* for firm j's variety k:

$$\bar{D}(P_{jk}) = \sigma f \left(\frac{\sigma}{\sigma - 1} c \right)^{\sigma - 1} P_{jk}^{-\sigma}. \tag{9}$$

Hence, because the MP-firms anticipate the adjustment in the SP-subsector, in the first stage these firms face the adjusted demand (9), which is independent from $\mathbb{P}$ and Y unlike the market demand (6). Indeed, the former encapsulates the reactions of the SP-firms to the MP-firms' actions, whereas the latter does not.

Since firm j's marginal cost is the same across its varieties and each variety is negligible within its product line, in equilibrium it must be that $P_{jk} = P_j$.[5] Therefore, this firm's profits may be rewritten as follows:

$$\Pi_j = n_j(P_j - C_j)\bar{D}(P_{jk}) - F_j = n_j(P_j - C_j)\sigma f \left(\frac{\sigma}{\sigma - 1} c \right)^{\sigma - 1} P_j^{-\sigma} - F_j$$

which depends *only* upon the common price P_j set by firm j. Hence, this firm's equilibrium price is given by

$$P_j^* = \frac{\sigma}{\sigma - 1} C_j.$$

This expression has the following far-reaching implication: *the MP-firms adopt the same pricing rule as the SP-firms*, the reason being that each big firm chooses its price schedule as if it were a monopolist facing its varieties' adjusted demands. Furthermore, when choosing its price, a MP-firm does not have to care about the equilibrium value of the total income because $\bar{D}(P_{jk})$ is independent from Y. Accordingly, despite the fact that big firms are atomic players, the presence of a competitive fringe is sufficient to trigger the disappearance of *all* forms of strategic interactions through the adjustment of the mass of small firms. This effect includes the interactions among big firms that affect the total price index, as well as the indirect interactions channeled by the redistribution of profits. Consequently, as long as the market accommodates a competitive fringe, we may conclude that both big and small firms adopt the same non-strategic market behavior. In contrast, once the entire competitive fringe has vanished, ($M^* = 0$), the big firms become strategic because both the price index and the total income vary with their price choices. To put it differently, we are back to the world of oligopoly theory. Note also that big firms' markups are lower in the presence of a competitive fringe than in a pure oligopolistic market. This is because (i) these firms behave like SP-firms, and (ii) monopolistic competition is here the limit of an oligopolistic market in which the number of firms is arbitrarily large. Since

[5] Hence, (2) may be viewed as a special case of (1) in which $n_j = 1$ for all j.

everything works as if the market would operate under monopolistic competition, markups are constant under CES preferences. In particular, markups are independent of the characteristics of the SP-subsector and of the number of MP-firms. Hence, we may conclude that *a competitive fringe stabilizes competition among big firms.*

The above discussion is summarized as follows.

Proposition 1. *When the market involves a fixed number of big firms and a competitive fringe governed by free entry and exit, all active firms adopt the same pricing rule. Furthermore, in equilibrium, each firm, either big or small, behaves like a monopolist facing its (adjusted) demand.*

To sum-up, once the value of M is endogenous, *the strategic interdependence among big firms is dissolved in an ocean of firms that all adopt the pricing behavior of small firms.* This provides a strong justification for the use of monopolistic competition in modeling imperfect competition since in the real world few sectors have a competitive fringe. Observe that increasing returns and product differentiation are needed for that to be true. Indeed, under constant or decreasing returns, the mass of small firms would be infinite when entry is free. Furthermore, firms' operating profits must be positive to cover their fixed costs. This shows how our setting differs from that of the dominant firm where small firms, the total mass of which is fixed, operate under decreasing returns and supply the same homogeneous good as the big firm.

The following two comments are in order. First, it is well known that the distinction between quantity or price competition is immaterial in monopolistic competition. In our setting, the market outcome is the same regardless of the strategy - quantity or price - used by big firms. In other words, Bertrand and Cournot competition yields the same market outcome. This strikingly differs from what we know in oligopoly theory and shows once more how a market involving big and small firms may obey very new rules. Second, unlike the oligopoly case, (9) implies that both the equilibrium price and profits of a big firm are independent from the size of the economy. In contrast, the size of the competitive fringe increases with the size of the economy.

It is tempting to believe that the above analysis is an artefact of the CES, which is known to yield fairly peculiar results. In addition, even though the big firms are heterogeneous, the small firms have been assumed to share the same marginal cost, while the empirical trade literature stresses the importance of firms' heterogeneity. We show below that the homogeneity assumption does not drive our main results, while we turn to the case of more general preferences in Section 5.

4 Heterogeneous firms

In this section, we assume that the SP-firms are heterogeneous in the sense of Melitz (2003). What makes our analysis potentially richer is that, in our setting, the cost cutoff could be influenced by the behavior of the MP-firms.

Prior to entry the SP-firms face uncertainty about their marginal cost. To enter the market, the SP-firms must bear a sunk cost f_e. After entry, the SP-firm i observes its marginal cost c_i, which is drawn randomly from the distribution $\Gamma(c)$, the density of which is denoted by $\gamma(c)$. When this firm decides to produce, it also incurs a fixed production cost f, so that its production cost function is given by $f + c_i q_i$. Under monopolistic competition, firm i sets a price equal to

$$p_i^* = \frac{\sigma}{\sigma - 1} c_i.$$

Since the equilibrium output is given by

$$q_i^* = \frac{Y}{\mathbb{P}^{1-\sigma}} \left(\frac{\sigma c_i}{\sigma - 1} \right)^{-\sigma} \tag{10}$$

the equilibrium profits are as follows:

$$\pi^*(c_i) = \frac{p_i^* q_i^*}{\sigma} - f = \frac{Y}{\sigma} \left(\frac{\sigma}{\sigma - 1} \frac{c_i}{\mathbb{P}} \right)^{1-\sigma} - f.$$

Because $\pi(c_i)$ is strictly decreasing in c_i, there is a unique cutoff $\bar{c}$ such that all firms having a marginal cost exceeding $\bar{c}$ decide not to produce and exit the market without incurring the fixed production cost f. Therefore, we may set $\pi(c) = 0$ for $c > \bar{c}$. The resulting truncated density of active firms is given by

$$\tilde{\gamma}(c) = \frac{\gamma(c)}{\Gamma(\bar{c})}$$

if $c \leq \bar{c}$ and $\tilde{\gamma}(c) = 0$ otherwise. Last, a firm enters when its expected profit net of entry cost is nonnegative. Therefore, if M_E firms enter, there will be $\Gamma(\bar{c}) M_E$ active firms.

Because all SP-firms face the same uncertainty, free entry implies that the expected profit is equal to zero in equilibrium:

$$\mathbb{E}_{\gamma}(\pi) = \int_0^{\infty} \pi(c)\gamma(c)\mathrm{d}c = \int_0^{\bar{c}} \pi(c)\tilde{\gamma}(c) \left(\Gamma(\bar{c})\mathrm{d}c \right) = \Gamma(\bar{c}) \mathbb{E}_{\tilde{\gamma}}(\pi) = f_e$$

which implies

$$\mathbb{E}_{\tilde{\gamma}}(\pi) = \frac{f_e}{\Gamma(\bar{c})} \equiv g(\bar{c}). \tag{11}$$

Let $\bar{p}$ and $\bar{q}$ be the equilibrium price and output of a firm having the cutoff marginal cost $\bar{c}$. Using (10), we have

$$\frac{\mathbb{E}_{\tilde{\gamma}}(p_i^* q_i^*)}{\bar{p}\bar{q}} = \frac{\mathbb{E}_{\tilde{\gamma}}(c_i^{1-\sigma})}{\bar{c}^{1-\sigma}}.$$

Since

$$\pi^*(c_i) = \frac{p_i^* q_i^*}{\sigma} - f$$

and

$$\frac{\bar{p}\bar{q}}{\sigma} = f$$

we can rewrite $\mathbb{E}_{\tilde{\gamma}}(\pi)$ as follows:

$$\mathbb{E}_{\tilde{\gamma}}(\pi) = \mathbb{E}_{\tilde{\gamma}}\left[\frac{p_i^* q_i^*}{\sigma} - f\right] = \left[\frac{\mathbb{E}_{\tilde{\gamma}}(c_i^{1-\sigma})}{\bar{c}^{1-\sigma}} - 1\right] f \equiv h(\bar{c}) \tag{12}$$

which depends only upon $\bar{c}$.

Inspecting (11) shows that $g(c)$ decreases with c from infinity to zero. Furthermore, Melitz (2003, p.1704) shows that $h(c)$ increases with c for most standard probability distributions. Hence, the value of $\bar{c}$ is uniquely determined by the intersection of the two loci g and h. Since the functions g and h are determined only by the properties of the density function γ and the parameters f and f_e, *the cutoff cost $\bar{c}$ is independent from the big firms' behavior.*

Using $\pi(\bar{c}) = 0$, we have

$$\frac{\bar{p}\bar{q}}{\sigma} = \frac{Y}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{\bar{c}}{\mathbb{P}}\right)^{1-\sigma} = f$$

which implies that the total price index is given by

$$\mathbb{P}^{1-\sigma} = \frac{Y}{\sigma f}\left(\frac{\sigma\bar{c}}{\sigma-1}\right)^{1-\sigma}.$$

which is identical to (8) but in which c is replaced by $\bar{c}$. Even though the total income depends on big firms' behavior through the redistribution of profits, we obtain a result identical to Melitz (2003).

Although the entrants make either positive or negative profits net of sunk costs f_E, the overall profits earned by the mass M_E of firms is zero. Indeed, the free entry condition implies

$$\mathbb{E}_{\gamma}(\pi) = f_E.$$

It then follows that

$$M_E f_E = \frac{M^*}{\Gamma(\bar{c})}\mathbb{E}_{\gamma}(\pi) = M^*\mathbb{E}_{\tilde{\gamma}}(\pi) = \int_0^{M^*} \pi^*(c_i)\mathrm{d}i.$$

Hence, as in Section 3, the total income is determined by the profits earned by the MP-firms only. Consequently, *small heterogeneous firms keep playing the role of a buffer.* This highlights how free entry and exit among SP-firms is the key-assumption in our analysis, as discussed in the preceding section.

To sum up,

Proposition 2. *When small firms are heterogeneous à la Melitz, Proposition 1 still holds.*

The equilibrium mass M_E^* of SP-firms that enter the market is determined by the behavior of the MP-firms, very much as M^* is determined in the homogeneous firm case. Since the share $1 - \Gamma(\bar{c})$ of active firms is constant, the mass of small firms that are active is given by $M^* = \Gamma(\bar{c})M_E^*$, thus implying that a shock on the MP-firms (e.g. the entry of a big firm) affect M_E^* and M^* proportionally.

5 General additive preferences

The purpose of this section is to generalize Proposition 1 to the case of general *additive* preferences:

$$\mathcal{U} = \int_0^M u(x_i)\mathrm{d}i + \sum_{j=1}^N \int_0^{n_j} U_j(X_{jk})\mathrm{d}k \tag{13}$$

where u and U_j are thrice continuously differentiable, strictly increasing and concave, with $u(0) = U_j(0) = 0$. Note that (13) allows for the utility of a SP-variety to differ from that of an MP-variety. The aim is to capture the idea that big and small firms may provide their customers with specific attributes. Hence, varieties do not enter symmetrically into individual preferences. Furthermore, besides the CES discussed in Section 2, another special case of (13) is given by the CARA utility proposed by Behrens and Murata (2007) to model preferences in monopolistic competition:

$$u(x) = 1 - \exp(-\alpha x) \qquad \alpha > 0.$$

Apart from the generalization of preferences, the model is the same as the one described in Section 2.

Let y be the consumer income defined in (3), P_j the price function set by the MP-firm j and p_i the price chosen by the SP-firm i. Denoting by λ the Lagrange multiplier, the utility-maximizing conditions yield the inverse demand functions:

$$p_i(x_i) = \frac{u'(x_i)}{\lambda} \qquad P_{jk}(X_{jk}) = \frac{U_j'(X_{jk})}{\lambda} \tag{14}$$

which means that the individual demand for any specific variety depends only upon its price and the marginal utility of income λ. Therefore, *the marginal utility of income captures all the market ingredients that matter to consumers* (and firms), very much as (4) in the Dixit-Stiglitz model. In addition, the strict concavity of u implies that the conditions (14) are sufficient for the consumer optimization problem to have a unique solution. As in the foregoing, the game is solved by backward induction.

Stage 2. As in Section 2, assume that the SP-firms share the same marginal cost c and fixed cost f. Therefore, firm i's profits are given by

$$\pi_i = \left[\frac{u'(x_i)}{\lambda} - c\right] Lx_i - f.$$

Using the"relative love for variety" $r_u(x_i)$ introduced by Zhelobodko et al. (2010):

$$r_u(x_i) \equiv -\frac{x_i u''(x_i)}{u'(x_i)}$$

we may state the equilibrium conditions in a very simple way. At a symmetric equilibrium, the first-order condition for profit-maximization may be written as follows:

$$p^* = \frac{c}{1 - r_u(x^*)}. \tag{15}$$

Combining this condition with the zero-profit condition yields the equilibrium output $q^* = Lx^*$, where x^* is the solution to

$$\frac{x^* r_u(x^*)}{1 - r_u(x^*)} = \frac{f}{cL}. \tag{16}$$

As shown by Zhelobodko et al. (2010), the equation (16) has a unique solution when (i) profits are concave ($r_{u'} < 2$), (ii) varieties are gross substitutes ($r_u < 1$) and (iii) the marginal revenue $((x_i u''(x_i) + u'(x_i))/\lambda)$ intersects once the horizontal line c for all admissible λ. Moreover, since the equations (15) and (16) do not depend on λ, *the decisions made by the SP-firms are independent from what the MP-firms do.*

Using (14) and (15), it is readily verified that the equilibrium value of the statistic λ is given by

$$\lambda^* = \frac{u'(x^*)[1 - r_u(x^*)]}{c} \tag{17}$$

which is independent from the MP-firms' actions. In other words, *when the big firms change their prices or outputs, only the mass of small firms is affected for the marginal utility of income to remain constant.* Furthermore, the marginal utility of income is independent from the big firms' actions. Thus, our setting may be viewed as a special aggregative game (Anderson et al., 2010) in oligopoly theory, in which the value of the marginal utility of income is the aggregate of the sole small players' actions.

Stage 1. Since the MP-firm j anticipates the equilibrium values of p^* and q^*, this firm is able to compute the resulting value of λ^* and to determine the adjusted inverse demand for its variety k:

$$P_{jk}(X_{jk}) = \frac{U_j'(X_{jk})}{\lambda^*}$$

which depends only upon the individual consumption of this variety since λ^* is already determined. In other words, all the properties shown in Section 3 remain valid.

Consequently,

Proposition 3. *Under general additive preferences, Proposition 1 still holds.*

In particular, when preferences are additive, the presence of a competitive fringe operating under free entry and exit is sufficient to make the big firms' market behavior non-strategic. Note also that Proposition 3 can be generalized to the case of heterogeneous firms à la Melitz, but additional conditions on the non-CES utility u must be imposed for the cutoff cost to be unique.

Stability in competition To better understand the role of the competitive fringe in stabilizing competition, consider the symmetric equilibrium $(\bar{p}, \bar{x})$ of the SP-subsector prevailing before a shock on big firms' costs and/or demands, and let $(\tilde{p}, \tilde{x})$ be the equilibrium prevailing after the shock. These two equilibria must satisfy the zero-profit condition:

$$\pi = [p(x) - c]Lx - f = 0.$$

Since the inverse demand
$$p(x) = u'(x)/\lambda$$
must be met in equilibrium, the shock to the MP-subsector may influence SP-firms' profits only through an upward (downward) shift of the inverse demand caused by a change in the value of λ. Hence, operating profits are increased (decreased) after the shock, which implies that the zero-profit condition cannot hold at $(\tilde{p}, \tilde{x})$. This means that λ must the same before and after the shock, which implies that the inverse demands for the MP-varieties are the same. When the mass of small firms remains the same after and before the shock, the value of the marginal utility of income is increased or decreased depending on the nature of the shock. Once small firms are free to enter or exit the market, their mass is determined by the zero-profit condition which restores the pre-shock value of λ.

We now want to stress that the above argument can be applied to more general settings. To be precise, consider an general preference model that generates small firms' demand schedules $d_i(p_i; \Lambda)$, where Λ is the vector of market statistics treated parametrically by small firms. For example, under additive preferences, we have $\Lambda = \lambda$. Additional examples are provided in the next section.

Let $\mathbf{a}$ be the vector of exogenous parameters affecting big firms' behavior, thus making Λ dependent on $\mathbf{a}$. Examples include parameters affecting big firms' cost (C_j) and demand (U_j) and number (N) only. Consider a shock on $\mathbf{a}$ which changes the vector $\Lambda(\mathbf{a})$ and assume that the shift of any small firm's demand is such that, everything else being equal, the pre-shock demand $d_i(p_i; \Lambda(\mathbf{a}))$ does not intersect the after-shock demand $d_i(p_i; \Lambda(\mathbf{a}'))$. Formally, this *no-crossing condition* may be stated as follows:

$$\Lambda(\mathbf{a}') \neq \Lambda(\mathbf{a}) => d_i(p_i; \Lambda(\mathbf{a})) > d_i(p_i; \Lambda(\mathbf{a}')) \quad \text{or} \quad d_i(p_i; \Lambda(\mathbf{a})) < d_i(p_i; \Lambda(\mathbf{a}')) \quad \text{for all } p_i > c.$$

Slightly extending the above argument shows that, under the no-crossing condition, the SP-demand functions remain the same for both the pre- and aftershock operating profits to be equal to f. Thus, we have:

Proposition 4. *Consider a preference structure such that the SP-demand schedules satisfy the no-crossing condition. Any exogenous shock affecting big firms' behavior triggers the entry or exit of small firms for the after-shock values of market statistics and small firms' prices to be equal to their pre-shock values.*

This proposition extends our main results to non-additive preferences compatible with the no-crossing condition. In particular, it implies that big firms do not behave strategically.

6 Quadratic utility

In this section, we consider a well-known example of *not* additively separable preferences. Specifically, the model is identical to the one presented in Section 2, except that individual preferences are described by the quadratic utility with cross-effects (Ottaviano et al., 2002):

$$\mathcal{U} = \mathbb{X} - \frac{\gamma}{2} \left(\int_0^M x_i^2 \mathrm{d}i + \sum_{j=1}^{N} \int_0^{n_j} X_{jk}^2 \mathrm{d}k \right) - \frac{\mathbb{X}^2}{2} \tag{18}$$

where the market statistic

$$\mathbb{X} = \int_0^M x_i \mathrm{d}i + \sum_{j=1}^{N} \int_0^{n_j} X_{jk} \mathrm{d}k$$

is the total consumption index, $\gamma > 0$ measuring the intensity of consumers' love for variety.[6]

First-order conditions for utility maximization with respect to each variety $i \in [0, M]$ and each variety $k \in [0, n_j]$ of firm $j = 1, ..., N$ yield the individual inverse demand functions:

$$\lambda p_i = 1 - \gamma x_i - \mathbb{X} \qquad \lambda P_{jk} = 1 - \gamma X_{jk} - \mathbb{X}.$$

where λ is again the marginal utility of income.

Using these conditions and the budget constraint, we can determined the equilibrium value of λ as follows:

$$\lambda^* = \frac{\gamma \left(\gamma + M + \sum n_j\right) Y - \gamma \mathbb{E}(\mathbf{P})}{\mathbb{E}^{\mathbf{2}}(\mathbf{P}) - \left(\gamma + M + \sum n_j\right) \mathbb{E}(\mathbf{P}^{\mathbf{2}})}$$

where $\mathbf{P}$ is the price profile of all varieties. Therefore, λ^* depends on the price moments $\mathbb{E}(\mathbf{P})$ and $\mathbb{E}(\mathbf{P}^2)$ as well as on the income Y. Since the MP-firms produce a strictly positive mass of varieties, λ^* can be impacted by the MP-firms' strategies chosen in the first stage through the prices' moments. On the contrary, the SP-firms will keep treating λ^* as given because they are negligible to the market.

After rearrangement, this yields the following expressions for the demand functions:

$$p_i = (1 - \mathbb{X} - \gamma x_i)/\lambda^* \qquad P_{jk} = (1 - \mathbb{X} - \gamma X_{jk})/\lambda^*. \tag{19}$$

Stage 2. Since each SP-firm is negligible, it accurately treats both $\mathbb{X}^*$ and λ^* as given. Therefore, the equilibrium output is given by

$$q_i^* \equiv q^* = \frac{L}{2\gamma}(1 - \mathbb{X} - \lambda^* c)$$

and the equilibrium price by

$$p_i^* \equiv p^* = \frac{1 - \mathbb{X} + \lambda^* c}{2\lambda^*}.$$

Conditional upon λ^*, the zero-profit condition determines the equilibrium value of the total consumption index $\mathbb{X}^*$:

$$\frac{L\lambda^*}{4\gamma}\left(\frac{1 - \mathbb{X}^*}{\lambda^*} - c\right)^2 - f = 0 \Rightarrow \mathbb{X}^* = 1 - \lambda^*\left(c + 2\sqrt{\frac{\gamma f}{\lambda^* L}}\right) \tag{20}$$

[6]Without loss of generality, the coefficients of $\mathbb{X}$ and $\mathbb{X}^2$ have been normalized to 1.

which depends on λ^*. It is worth stressing the difference this setting and the one presented in the previous sections. The marginal utility of income is *not* anymore determined by the SP-firms only because it now depends on the actions chosen by all (big and small) firms through $\mathbb{X}^*$.

Stage 1. The MP-firms face the inverse demand given by

$$P_{jk}[X_{jk}; \mathbb{X}^*(\lambda^*)] = \frac{1 - \mathbb{X}^*(\lambda^*) - \gamma X_{jk}}{\lambda^*}.$$

Because the MP-firm j accurately anticipates the value of $\mathbb{X}^*$, the adjusted demand for its variety k is obtained as follows:

$$P_{jk}(X_{jk}; \lambda^*) = 2\sqrt{\frac{\gamma f}{\lambda^* L}} - \frac{\gamma}{\lambda^*} X_{jk}.$$

Since every MP-firm j influences the value of λ^* through the price moments, this firm understands it can manipulate λ^* in the first stage game. Therefore, big firms' actions affect their adjusted demands, which implies that these firms compete strategically. In this case, their actions also affect the small firms' choices, not just their mass. This example shows that Proposition 1 need not hold under non-additive preferences, thus highlighting the role of additive preferences in stabilizing competition in markets with big and small firms.

However, things become very different when (18) is embedded into a quasi-linear upper-tier utility:

$$\mathcal{U} = \mathbb{X} - \frac{\gamma}{2}\left(\int_0^M x_i^2 \mathrm{d}i + \sum_{j=1}^{N} \int_0^{n_j} X_{jk}^2 \mathrm{d}k\right) - \frac{\mathbb{X}^2}{2} + H$$

where H is the numéraire, very much like in several partial equilibrium analysis. If each consumer is endowed with a quantity of the numéraire sufficiently large for the equilibrium consumption of this good to be positive, the marginal utility of income is fixed and equal to 1. As shown by (20), the equilibrium value $\mathbb{X}^*$ is constant and *independent* from the choices made by the MP-firms. Consequently, Proposition 1 holds true in the linear-quadratic utility case. This is because the marginal utility of income is fixed ($\lambda = 1$), while a firm's demand is shifted upward or downward with the total output ($\Lambda = \mathbb{X}$). Thus, the no-crossing condition is guaranteed.

By way of contrast, in the quadratic case (18), we have seen that λ and $\mathbb{X}$ are interdependent so that $\Lambda = (\lambda, \mathbb{X})$. Would an external shock shift λ and $\mathbb{X}$ in the same direction, the no-crossing would hold. However, (20) shows that they move in opposite directions, thus showing why Proposition 1 does not hold under (18).

The left and right panels of Figure 1 illustrate the differences between the quadratic and linear-quadratic cases, respectively. In the former one, (19) shows that a simultaneous change in λ and $\mathbb{X}$ modifies demands through a rotation and a shift of the corresponding curves. This changes both the slope and the intercept of the demand schedules, thus

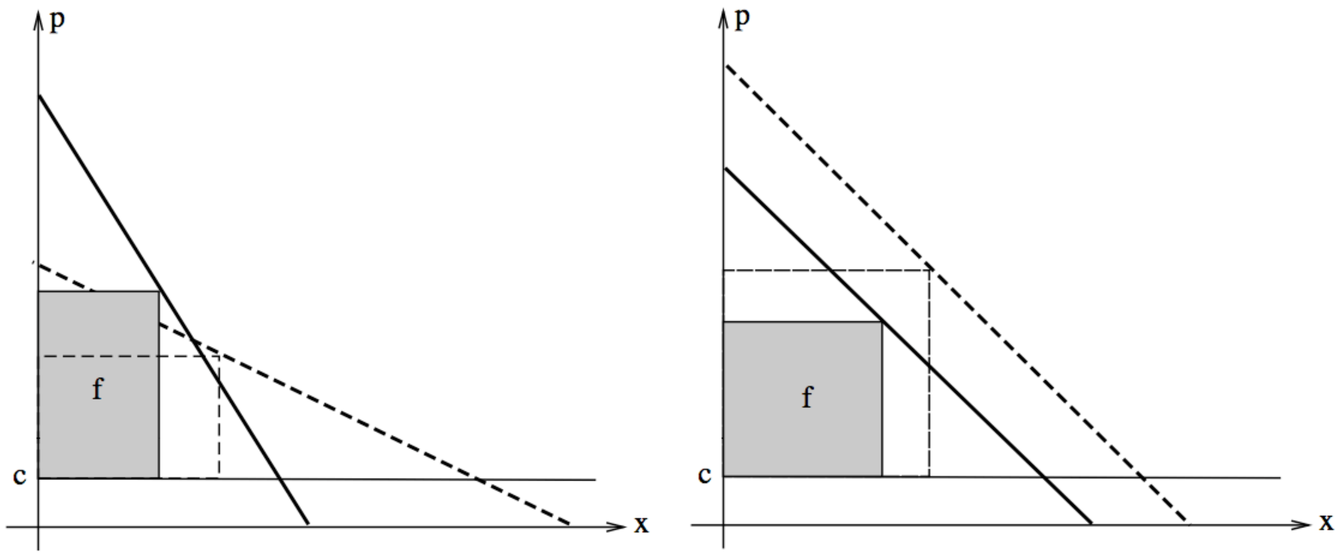

Figure 1: Equilibria without and with price stabilization

violating the no-crossing property. In the latter one, λ is fixed so that a change in $\mathbb{X}$ triggers an upward or downward parallel shift of the demand schedule. Hence, only the intercept of demands is modified, which prevents the pre-shock and post-shock curves to intersect.

The above discussion has an important methodological implication: modeling competition in an industrial sector that has a competitive fringe by means of a pure oligopoly setting may yield dubious results.

7 Concluding remarks

We would be the last to claim that oligopoly theory is irrelevant. Instead, our aim was to show that differentiated markets involving a few big firms endowed with market power as well as a competitive fringe may be accurately described by monopolistic competition settings. In particular, when the no-crossing condition holds, small firms' equilibrium price and output are stable in response to any shock affecting big firms. Furthermore, when preferences for the differentiated good are additive, (i) the marginal utility of income captures all the market ingredients that matter to consumers and firms; (ii) any firm, either big or small, either homogeneous or heterogeneous, adopts a non-strategic market behavior, meaning that its profit-maximizing strategy is independent from the actions taken by its competitors; and (iii) when firms are heterogeneous, the average productivity within the competitive fringe is independent from the big firms' behavior.

To conclude, observe that our analysis highlights the fact that small firms enter or exit the market in response to an external shock in a way such that big firms have no incentives to deviate from the strategies adopted before the shock. Thus, we shed new light on the well-documented stickiness of prices on several markets.

References

[1] Anderson, S.P., E. Nisvan and D. Piccinin (2010) Aggregative games with entry. University of Virginia, memo.

[2] Aumann, R.J. (1964) Markets with a continuum of traders. *Econometrica* 32, 39-50.

[3] Behrens, K and Y. Murata (2007) General equilibrium models of monopolistic competition: a new approach. *Journal of Economic Theory* 136, 776-787.

[4] Bernard, A., J. Jensen, S. Redding, and P. Schott (2007) Firms in international trade. *Journal of Economic Perspectives* 21(3), 105-130.

[5] Dixit, A.K. and J.E. Stiglitz (1977) Monopolistic competition and optimum product diversity. *American Economic Review* 67, 297-308.

[6] Houthakker, H.S. (1960) Additive preferences. *Econometrica* 28, 244-257.

[7] Kydland, F. (1979) A dynamic dominant firm model of industry structure. *The Scandinavian Journal of Economics* 81, 355-366.

[8] Markham, J.W. (1951) The nature and significance of price leadership. *American Economic Review* 41, 891-905.

[9] Melitz, M. (2003) The impact of trade on intraindustry reallocations and aggregate industry productivity. *Econometrica* 71, 1695-1725.

[10] Neary, J.P. (2009) International trade in general oligopolistic equilibrium. University of Oxford, memo.

[11] Neary, J.P. (2010) Two and a half theories of trade. *The World Economy* 33, 1-19.

[12] Ottaviano, G.I.P., T. Tabuchi, and J.-F. Thisse (2002) Agglomeration and trade revisited. *International Economic Review* 43, 409-436.

[13] Parenti, M. (2010) Trade under oligopolistic competition in the presence of a competitive fringe. Université de Paris-I, memo.

[14] Shimomura, K.-I. and J.-F. Thisse (2009) Competition among the big and the small. CEPR Discussion Paper No.7404.

[15] Zhelobodko, E. S. Kokovin and J.-F. Thisse (2010) Monopolistic competition: beyond the CES. CEPR Discussion Paper No.7947.