On Price, Entry and Market Size. A General Monopolistic Competition Approach*

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Abstract

We propose a general model of monopolistic competition and derive a complete characterization of the market equilibrium based on an Arrow-Pratt measure of concavity of an otherwise unspecified utility function. In particular, we show that any utility belonging to one of two classes of functions generate opposite market effects, the CES being the borderline case. In particular, when the relative love for variety increases with the individual consumption level, the market displays standard competitive effects. On the contrary, when the relative love for variety decreases, the equilibrium price increases with the number of firms and the market size. Finally, we apply our setting to trade theory and uncover several new properties hindered by the CES, such as dumping and reverse dumping. We also derive several general properties of the trade pattern.

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1 Introduction

Monopolistic competition has been used successfully in a wide range of fields, including growth and development, international trade, and economic geography (Matsuyama, 1995; Brakman and Heijdra, 2004). It is also one of the key-stones of the new and fast-growing literature initiated by Melitz (2003) on firms' heterogeneity. Although the CES utility model developed by Dixit and Stiglitz (1977) has been the workhorse of the vast majority of contributions using monopolistic competition, it seems fair to say that this model suffers from several major drawbacks.[1] First, the individual consumption pattern lacks flexibility since the elasticity of substitution is constant and the same across varieties. Second, the market outcome is not directly affected by the entry of new firms. In particular, markups are independent of the number of competitors. This runs against empirical evidence, which shows that firms operating in bigger markets typically have lower markups (Syverson, 2007). Third, there is no scale effect, that is, the size of firms is independent of the number of consumers, which contradicts the fact that firms tend to be larger in larger markets (Campbell and Hopenhayn, 2005). The size of the economy affects only the number of operating firms through free entry and exit. Fourth, due to its very specific nature, the CES model yields fairly particular results whose robustness is hard to check. Last, in many applications, the CES utility is nested into an upper-tier Cobb-Douglas utility. This implies that expenditures shares and demands for different types of goods are independent from each other. To a large extent, these simplifying assumptions explain the success of the Dixit-Stiglitz model: the CES provides an operationally analytical tool that can be used as building-blocks in various settings.

Thus, we find it both meaningful and important to develop a general model of monopolistic competition. It must admit the Dixit-Stiglitz setting as a special case in order to assess the impact of the various restrictions imposed by this model. To provide a better description of the working of markets, this model must also cope with the main issues highlighted in oligopoly theory.[2] Developing such a general model and studying the properties of the market

[1] Dixit and Stiglitz (1977) starts with a fairly general setting but most of their results are derived for a symmetric CES utility.

[2] In this respect, it is worth stressing that reducing the gap between monopolistic competition and industrial

equilibrium is precisely the main objective of this paper.

To achieve our goal, we follow Dixit and Stiglitz (1977) and assume that preferences over the differentiated product supplied by monopolistically competitive (hereafter, MC-) firms are additively separable across varieties. However, unlike Dixit and Stiglitz who work mainly with a power function, we derive the properties of the market outcome for a general and unspecified utility function. Even though varieties enter utility in a symmetric way, this allows us to describe various patterns of substitution through the *relative love for variety*, which is the counterpart of the relative measure of risk aversion à la Arrow-Pratt. For example, in the case of two varieties, the degree of convexity of the indifference curves toward the origin may be viewed as a measure of the intensity of consumers' love for variety. When these curves are straight lines, consumers consider all varieties as perfect substitutes. At the other extreme, when consumers have Leontief preferences, varieties are perfect complements: consumers maintains the same relative share of the two varieties. This formal analogy will permit us to exploit the various properties of risk-aversion theory in studying the properties of monopolistic competition. In particular, it will open to the door to alternatives to the CES, the counterpart of which is a utility displaying a constant relative risk aversion.[3]

Although our setting retains the flexibility of the Dixit-Stiglitz model by ignoring strategic interactions, thereby remaining compact and tractable, it is sufficiently rich (i) to display a wide range of effects highlighted in industrial organization and (ii) to uncover new and unsuspected results under well-behaved utility functions. Specifically, we will show that *the market outcome depends on how the relative love of variety varies with the consumption level*, which in turn determines the price-elasticity of demands. To be more precise, firms' demands become more (resp., less) inelastic when the relative love for variety increases (resp., decreases). This is because consumers' preferences for more balanced bundles of varieties are stronger (resp., weaker). **To get further insight, we show that at the symmetric outcome, the relative love for variety is equal to the elasticity of substitution across varieties. Unlike the CES in which the elasticity of substitution is given a priori, here it varies with the individual consumption

organization has been the main motivation for using the quadratic, trans-log and CARA utilities (Ottaviano et al., 2002; Feenstra, 2003; Behrens and Murata, 2007).

[3]Behrens and Murata (2007) have proposed to use the constant absolute risk aversion utility.

level.** Depending on the properties of the relative love for variety or, equivalently, of demands' price-elasticity, we will see that the market outcome may obey two opposite patterns. On the one hand, when the relative love for variety increases with the consumption of each variety, the equilibrium displays the standard *pro-competitive* effects generated by the entry of new firms and a larger size of the market, two effects that the CES does not apprehend: more firms, a larger market size, or both lead to *lower* market prices. On the other hand, when the relative love for variety decreases, the equilibrium displays an *anti-competitive* behavior, meaning that the entry of new firms, a larger market, or both lead to *higher* prices. Although at variance with the standard paradigm of entry, this result agrees with several contributions in product differentiation theory (Deneckere and Rothschild, 1992; Chen and Riordan, 2007, 2008). We must stress here that the CES is the dividing line between these two classes of utility functions since it does not display any of these effects. Furthermore, though the mass of firms always increases with the size of the market, it does so less than proportionally in the pro-competitive case and more than proportionally in the anti-competitive one. This is due to the above-mentioned difference in the price effect.

Our research strategy is also appealing from the empirical point of view because it allows one to determine necessary and sufficient condition on utilities for the equilibrium to display a pro- or anti-competitive behavior. Therefore, the specification of a particular utility needed for estimation sparks automatically the underlying market properties. Another by-product of our research strategy is that our results still hold in a multi-sector economy under fairly mild assumptions. This has the following implication: *though most sectors of the economy are probably pro-competitive, this does not preclude the existence of a few anti-competitive ones.* Or, to put it differently, that entry may lead to higher prices in some specific markets need not be viewed as being odd.

In order to show the relevance of our modeling strategy for various economic fields, we apply our setting to new trade theory. Specifically, we show that firms' pricing behavior may exhibit (reciprocal or unilateral) dumping (Brander and Krugman, 1983) or (reciprocal or unilateral) reverse dumping (Greenhut *et al.*, 1987), depending on the nature of the utility and the relative sizes of the trading partners.[4] We also derive properties of the consumption and production patterns that are independent of the utility function. In particular, consumers living in the bigger country buy less of the domestic varieties than of the foreign varieties.

Before proceeding, we must stress that our model bears several similarities with Krugman (1979). Yet, Krugman did not explore the market implications of his model, perhaps because his purpose was different from ours. His approach has been ignored in subsequent works. As observed by Neary (2004, p.177), this is probably because Krugman's specification of preferences "has not proved tractable, and from Dixit and Norman (1980) and Krugman (1980) onwards, most writers have used the CES specification." Instead, we show that Krugman's approach is *tractable*. To be precise, by using the elasticity of the marginal utility, we can provide a complete characterization of the market outcome and of all the comparative statics implications in terms prices, consumption level, outputs, and mass of firms/varieties.

The paper is organized as follows. The next section presents and develops the model in the case of a single sector. Specifically, we characterize the short-run equilibrium in which the mass of firms is exogenous and the long-run equilibrium in which the mass of firms is determined by free entry. Section 3 deals with the case of a two-sector economy, one being monopolistically competitive and the other perfectly competitive. In section 4, we apply our approach to a new trade theory setting and derive the properties of the trade pattern and firms' pricing behavior. Section 5 concludes.

2 The one-sector economy

2.1 The model

The economy involves one MC-sector supplying a differentiated good and one production factor, labor. There are L workers who supply each E units of labor to the MC-sector. Labor is chosen as the numéraire so that E stands for both worker income and expenditure.

The output of the MC-sector is made available as a continuum N of horizontally differen-

[4]See, e.g. Martin (2009) for empirical evidence and references.

tiated varieties indexed by $i \in [0, N]$. Preferences are additively separable and given by

$$\mathcal{U} = \int_0^N u\left[x(i)\right] \mathrm{d}i$$

where $x(i)$ denotes the consumption of variety i, while $u(\cdot)$ is a thrice continuously differentiable, strictly increasing and strictly concave function. The representative consumer maximizes her utility $\mathcal{U}$ subject to the budget constraint

$$\int_0^N p(i)x(i)\mathrm{d}i = E$$

where $p(i)$ is the price of variety i. Though i varies continuously in the interval $[0, N]$, it is notationally convenient to use i as an index of p and x.

The assumptions made on utility u imply that a consumer displays a *love for variety.* Let indeed $Q > 0$ be any given total quantity of the differentiated good. If this consumer gets the same number X/M units of each variety $i \in [0, Q/M]$ with $M < N$, she enjoys the utility level $\mathcal{U}(M; Q) = Mu(Q/M) + (N - M)u(0)$. Note that $u(0) \neq 0$ implies that increasing the number of varieties affects the consumer's well-being even when she does not change her consumption pattern. This does not strike us as being plausible. For this reason, we assume from now on that $u(0) = 0$. That said, it is readily verified that $Mu(Q/M)$ is a strictly increasing function of M under the assumptions made on u. Consequently, rather than concentrating her consumption over a small mass of varieties, the consumer prefers to spread it over a larger mass of varieties until M is equal to the total mass N of varieties available. This implies that our setting does not impose any additional restriction on u for consumers to exhibit a love for variety.

All of this has the following major implication: *individual consumption in the theory of monopolistic competition with love for variety is formally equivalent to individual decision-making in the Arrow-Pratt theory of risk aversion*, the mixture of risky assets being replaced with the mixture of differentiated varieties. This will allow us to derive properties of firms' demands that are both intuitive and simple.

To determine the equilibrium consumption, we differentiate the Lagrangian

$$\mathcal{U} + \lambda \left[E - \int_0^N p_i x_i \mathrm{d}i \right]$$

with respect to x_i and get

$$u'(x_i) = \lambda(\cdot) p_i \tag{1}$$

where the Lagrange multiplier $\lambda(\cdot)$ depends on the price function $p(\cdot)$, the mass of varieties N, and the expenditure level E. In other words, *the marginal utility of income*, $\lambda(\cdot)$, *captures all market ingredients that matter to firms*, very much as the price index does in the Dixit-Stiglitz model. It also comprehends the type and nature of competition across firms through its behavior with respect to the structure of individual preferences.

Setting $\varphi \equiv (u')^{-1}$, we obtain the individual demand for variety i:

$$x_i = \varphi[\lambda(\cdot) p_i].$$

Since the market demand is given by Lx_i, L plays the role of a scaling factor. Unlike Dixit and Stiglitz (1977), or Behrens and Murata (2007), we do not assume that u, hence φ, takes a specific functional form. Instead, we will keep u unspecified. Since $\varphi' = 1/u'' < 0$, x_i is a strictly decreasing function of p_i.

Each firm produces a single variety and any two firms do not sell the same variety. Because there is a continuum of firms, each firm is negligible to the market. Hence, *it may accurately treat the Lagrange multiplier* $\lambda(\cdot)$ *as a parameter*. In other words, the price choice made by any other firm $j \neq i$ has no impact on firm i's demand function:

$$\frac{\partial x_i}{\partial p_j} = 0 \qquad \text{for all } j \neq i.$$

Consequently, the functional form of the price-elasticity of firm i's demand

$$\varepsilon_i(p_i) \equiv -\frac{p_i}{x_i} \frac{\partial x_i}{\partial p_i}$$

is determined only through the properties of the utility u. Therefore, the demand for variety i depends on its price p_i and on the marginal utility of income. Since all firms face the same multiplier, the functional form of the demand is the same across firms. Similarly, the individual inverse demand

$$p_i(x_i) = u'(x_i)/\lambda(\cdot) \tag{2}$$

is uniquely determined and strictly decreasing in x_i. Since the Lagrange multiplier acts only as a scaling factor, (2) implies that *the inverse demand and the marginal utility have the same properties.*

We now show how the profit-maximizing conditions can be expressed through a simple measure of the concavity of u. The first-order conditions for utility maximization implies that

$$u'(x_i) = u'(x_j)\frac{p_i}{p_j}. \tag{3}$$

Since $\varphi \equiv (u')^{-1}$, we get:

$$x_i = \varphi\left[\varphi^{-1}(x_j)\frac{p_i}{p_j}\right]. \tag{4}$$

Differentiating (4) with respect to p_i yields

$$\begin{aligned} \frac{\partial x_i}{\partial p_i} &= \frac{\partial}{\partial p_i}\varphi\left[\varphi^{-1}(x_j)\frac{p_i}{p_j}\right] = \varphi'\left[\varphi^{-1}(x_j)\frac{p_i}{p_j}\right]\cdot\left[\left(\varphi^{-1}\right)'(x_j)\frac{p_i}{p_j}\frac{\partial x_j}{\partial p_i} + \frac{\varphi^{-1}(x_j)}{p_j}\right] \\ &= \varphi'(\varphi^{-1}(x_i))\frac{\varphi^{-1}(x_i)}{p_i} = \frac{u'(x_i)}{p_i u''(x_i)} = -\frac{x_i}{r_u(x_i)p_i} \end{aligned} \tag{5}$$

where we have used the property $\varphi' = 1/u''$, (3), and the equality $\partial x_j/\partial p_i = 0$. In this expression,

$$r_u(x_i) \equiv -\frac{x_i u''(x_i)}{u'(x_i)} > 0 \tag{6}$$

is the *relative love for variety* (hereafter, RLV). As will be seen below, $r_u(x_i)$, which is a local measure of the intensity of love for variety, is *the* key concept for our study of monopolistic competition. In the CES case, we have

$$u(x_i) = x_i^{\rho}$$

where ρ is a constant such that $0 < \rho \leq 1$, which implies a constant RLV:

$$r_u(x_i) = 1 - \rho.$$

Behrens and Murata (2007) retain the CARA utility $u(x) = 1 - \exp(-\alpha x)$ where $\alpha > 0$ is the absolute love for variety, so that the corresponding RLV, i.e. $r_u = \alpha x$, increases with the consumption level.

To better understand the economic meaning of the relative love for variety, it turns out to be useful to evaluate the RLV along the diagonal in the quantity space ($x_i = x$). Using Nadiri (1982, p.442), we obtain

$$r_u(x) = \frac{1}{\sigma(x)}. \tag{7}$$

In this special case, *the RLV is the inverse of the elasticity of substitution across varieties.*[5] Unlike the CES where the elasticity of substitution is exogenous and constant, the value of σ varies here with the consumption level x. As will be shown below, the equilibrium is determined via two effects, i.e. a quantity effect and a differentiation effect. More precisely, when they consume a larger quantity of the differentiated product, (7) implies that consumers perceive varieties as being more (resp., less) differentiated if and only if the RLV increases (resp., decreases) with the consumption level. Formally, this means that, as we move along the diagonal the indifference curves become more (resp., less) radially convex.[6] These two patterns are depicted in Figure 1. Both seems a priori relevant, which means that it is hard to make predictions about the behavior of the RLV.

[5] Off-diagonal, the expression of the elasticity of substitution is much more involved so that (7) does not hold anymore.

[6] Consider two positive and decreasing functions $f_1(x)$ and $f_2(x)$ and let $x_i(\alpha)$ be the solution to $f_i(x_i) = \alpha x_i$ for $\alpha \in (0, \pi/2)$ so that α and x_i move in opposite directions. We say that f_1 is *more radially convex* that f_2 if $d[f_1'(x_1(\alpha))/f_2'(x_2(\alpha))]/d\alpha > 0$ holds for all $\alpha \in (0, \pi/2)$. In words, the curvature of f_2 is stronger than that of f_1 when the ratio of the slopes of the two functions increases with the slope of the radius.

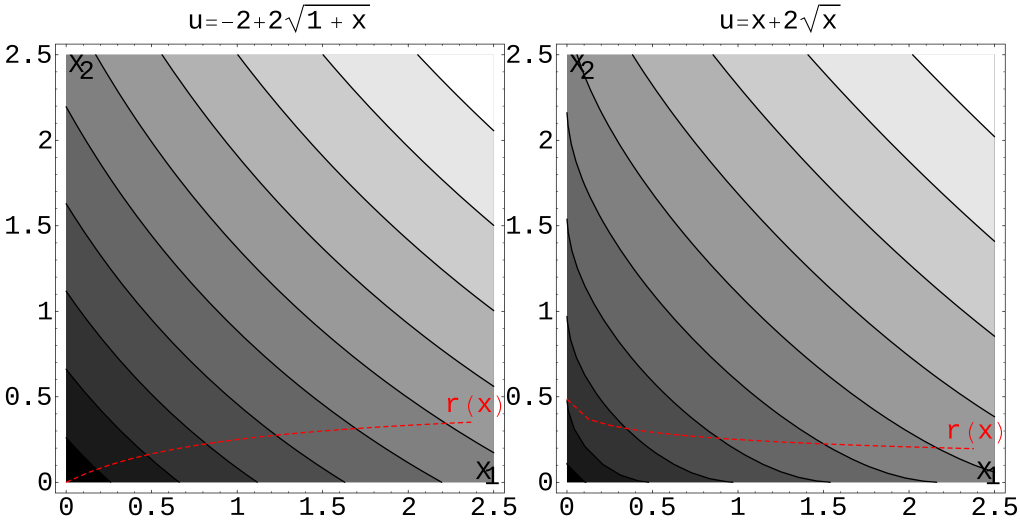

Figure 1: Indifference curves of $u(x_1) + u(x_2)$ for two varieties x_1, x_2 under pro- and anti-competitive utilities.

Furthermore, (5) implies that the price-elasticity of the demand for variety i may be rewritten as follows:

$$\varepsilon_i(p_i) = \frac{1}{r_u(x_i(p_i))}. \tag{8}$$

Therefore, *a stronger* (resp., weaker) *love for variety generates less* (resp., more) *elastic demand functions.* This is because a stronger love for variety induces consumers to focus more on a balanced mix of varieties, which in turn makes the demands for these two varieties less sensitive to changes in their relative prices. Since we can work indifferently with the demand or the inverse demand, we have

$$e_i(x_i) \equiv -\frac{x_i}{p_i}\frac{\partial p_i}{\partial x_i} = r_u(x_i). \tag{9}$$

where e_i is the elasticity of firm i's inverse demand function p_i.

Finally, to undertake production, every firm needs a fixed requirement $f > 0$ and a marginal requirement $c > 0$ of labor. Since labor is the numéraire, the production cost of the firm supplying the quantity q is given by $C(q) = f + cq$. The profit function of the firm supplying variety i is then given by

$$\begin{aligned}\pi(p_i; p(\cdot), E) &= (p_i - c)Lx_i - f \\ &= (p_i - c)L\varphi[\lambda(\cdot)p_i] - f.\end{aligned}$$

Because each firm treats $\lambda(\cdot)$ parametrically, it behaves like a monopolist on its market. Hence, maximizing profits with respect to price or quantity yields the same equilibrium outcome. In this case, the profit function may be rewritten as follows:

$$\pi(x_i; x(\cdot), E) = \left[\frac{u^{'}(x_i)}{\lambda(\cdot)} - c\right] Lx_i - f.$$

For any given value of $\lambda(\cdot)$, this function has a maximizer if the following two conditions hold:

$$\lim_{z\to 0}[u^{'}(z) + zu''(z)] = \infty \qquad \lim_{z\to\infty}[u^{'}(z) + zu''(z)] \leq 0. \tag{10}$$

In other words, the value of the marginal revenue is high when x_i is small and low when x_i is large, thus intersecting at least once the horizontal line $\lambda(\cdot)c$. Moreover, this maximizer is unique if $\pi(x_i; x(\cdot), E)$ is strictly concave with respect to x_i. Expressed in terms of the utility u, it is readily verified that this condition is equivalent to

$$r_{u'}(x_i) = -\frac{x_i u'''(x_i)}{u''(x_i)} < 2. \tag{11}$$

Throughout the rest of this paper, we assume that the conditions (10)-(11) always hold.

Observe that the strict concavity of profits means that the marginal revenue is strictly decreasing. Consequently, for a given utility u, two cases may arise. First, the equation $u^{'}(z) + zu''(z) = 0$ has no solution. Using (10), it must be that $r_u(z)$ is smaller than 1 for all $z > 0$. Second, the equation $u^{'}(z) + zu''(z) = 0$ has a solution z_0. Then, we have $r_u(z) < 1$ for $z < z_0$ and $r_u(z) > 1$ for $z > z_0$. In this case, we may restrict ourselves to the interval $(0, z_0)$ in which $r_u(z) < 1$ since all equilibria belong to $(0, z_0)$. Without loss of generality, we may then assume that $r_u(z) < 1$ for all relevant values of $z > 0$. In what follows, we will assume a somewhat stronger condition:

$$r_u(z) < 1 \qquad \text{for all } z \geq 0. \tag{12}$$

In order to disentangle the various effects at work, it is both relevant and convenient to distinguish between what we call a *short-run equilibrium*, in which the mass N of firms is fixed and a *long-run equilibrium* in the which the mass of firms is endogenously determined through free entry and exit. This distinction is of particular importance when assessing the impact of entry and market size on the market outcome.

2.2 The short-run equilibrium

Given the mass N of firms, a short-run equilibrium is a price function $\mathbf{p} = p_{i \leq N}$ such that no firm finds it profitable to deviate unilaterally.

Following the literature, we focus on a symmetric equilibrium, $p_i = \bar{p}(N)$, and use (8) to write the first-order condition for firm i's profit maximization as follows:

$$\bar{M} \equiv \frac{\bar{p} - c}{\bar{p}} = r_u(\bar{x}) \tag{13}$$

where $\bar{x} = x_i(\bar{p})$ is the equilibrium consumption of each variety $i \in [0, N]$. Hence, in equilibrium, *the mark-up of a firm is equal to the RLV*. Since $r_u(\bar{x}) < 1$, the elasticity of substitution must exceed 1 at any symmetric equilibrium. Furthermore, since all firms face the same Lagrange multiplier $\lambda(\cdot)$, the solutions to the first-order condition are the same across firms, which implies that all solutions (if any) are such that all prices are equal. Hence, if a short-run equilibrium exists, it is unique and symmetric.

At any price $\bar{p}$, the consumption level is the same across varieties and equal to $\bar{x} = E/N\bar{p}$. The equilibrium condition (13) thus becomes

$$1 - \frac{c}{\bar{p}} = r_u\left(\frac{E}{N\bar{p}}\right). \tag{14}$$

Both sides of this expression are continuous. Furthermore, since $l_1(p) \equiv 1 - c/p$ increases from

0 to 1 on $[c, \infty)$, it intersects $l_2(p) \equiv r_u(E/Np)$ at some $p > c$ if the two conditions $l_2(c) > 0$ and $l_2(\infty) < 1$ hold. The latter condition is equivalent to (12). Under this condition together with (10) and (11), a symmetric and unique short-run equilibrium always exists.

Differentiating the equilibrium condition (14) with respect to N leads to the expression

$$\left(c + \frac{E}{N} r_u'\right) \frac{d\bar{p}}{dN} = -\frac{\bar{p}E}{N^2} r_u'$$

which can be rewritten as follows:

$$\left(\frac{c}{\bar{p}} + \frac{E}{N\bar{p}} r_u'\right) \frac{N}{\bar{p}} \frac{d\bar{p}}{dN} = -\frac{E}{N\bar{p}} r_u'.$$

Plugging the solution of (14) with respect to $c/\bar{p}$ and $\bar{x} = E/N\bar{p}$ into the above expression yields

$$(1 - r_u + \bar{x} r_u') \frac{N}{\bar{p}} \frac{d\bar{p}}{dN} = -\bar{x} r_u'. \tag{15}$$

Differentiating (6) with respect to x_i, we obtain the identity:

$$\bar{x} r_u' = (1 + r_u - r_{u'}) r_u. \tag{16}$$

Substituting for $\bar{x} r_u'$ into the left-hand side of (15), we get

$$1 - r_u + \bar{x} r_u' = 1 + r_u^2 - r_u r_{u'} > (1 - r_u)^2 \geq 0$$

where we have used (11). Therefore, (15) implies that $d\bar{p}/dN$ and $r_u'(\bar{x})$ have opposite signs. Consequently, the following three cases may arise.

First, if $r_u'(\bar{x}) > 0$, it must be that

$$\frac{d\bar{p}}{dN} < 0.$$

In words, if the RLV increases with the consumption level or, equivalently, if the elasticity of substitution decreases, the entry of new firms in the MC-sector leads to a lower market price.[7] This is the standard *pro-competitive* effect generated by the entry of new firms. Clearly, the condition $r'_u > 0$ is also necessary for this effect to hold true.

Second, if $r'_u(\bar{x}) < 0$, we have

$$\frac{d\bar{p}}{dN} > 0.$$

In other words, when the RLV decreases with the consumption level, the entry of firms into the MC-sector leads to a higher market price, an effect that may be described as being *anti-competitive*. Again, the condition $r'_u(\bar{x}) < 0$ is necessary for the anti-competitive effect to arise.

Last, if $r'_u(\bar{x}) = 0$, then we have

$$\frac{d\bar{p}}{dN} = 0$$

which means that increasing the mass of firms has no impact on the equilibrium market price. Hence, the elasticity of substitution across varieties must vary with the consumption level for the market price to be affected by the entry of new firms.

The above discussion is summarized as follows.

Proposition 1 *If (10)-(12) hold, then there exists a unique and symmetric short-run equilibrium. Furthermore, when the relative love for variety increases (resp., decreases) with the consumption level, then the equilibrium price decreases (resp., increases) with the mass of firms. The equilibrium price is independent of the mass of firms if and only if the utility is given by a CES.*

The CES is the only function that has a constant RLV. It is, therefore, the only utility for which entry does not impact on the equilibrium price. Hence, we may safely conclude that the CES is the borderline between two general, but very different, classes of utility functions. This in turn shows how peculiar is the CES for the study of monopolistic competition. In addition,

[7] We have seen that the CARA-utility has an increasing relative love for variety. So, it is no surprise that Behrens and Murata (2007) prove that the market outcome displays a pro-competitive behavior.

we will see below that both the pro- and anti-competitive outcomes may be generated by well-behaved utility functions.

To understand the economic intuition behind the pro- and anti-competitive effects, it is convenient to work with the indirect demand function $p(x)$. The expression (9) shows that $r_u' > 0$ amounts to saying that the elasticity of the inverse demand is increasing, whereas $r_u' < 0$ means that this elasticity is decreasing. Let us now assume that N increases, while the market price $\bar{p}$ remains the same. Because consumers display a love for variety, the supply of more varieties induces them to spread their consumption over a wider range of varieties, thus reducing to $\tilde{x} < \bar{x}$ the individual consumption of each variety. Since u is strictly concave, it must be that $u'(\tilde{x}) > u'(\bar{x})$. It then follows from $u^{'}(\tilde{x}) = \tilde{\lambda}\bar{p}$ and $u^{'}(\bar{x}) = \bar{\lambda}\bar{p}$ that $\tilde{\lambda} > \bar{\lambda}$. In other words, because there are more varieties the marginal utility of income increases with N. Consequently, the inverse demand for each variety is shifted downward.

However, when r_u is increasing or decreasing, entry does affect the market price. It is well known that the first-order condition for profit maximization is given by

$$p_i[1 + e_i(x_i; N)] = c.$$

When $e_i(x_i; N)$ increases with N, the above expression shows that entry yields a lower market price. In contrast, when $e_i(x_i; N)$ is decreasing, entry leads to a higher market price. This contrast in results depends on the way the slope of the inverse demand $p(x)$ changes in the neighborhood of the equilibrium, hence how its elasticity varies.

To illustrate those results, it is convenient to focus on a class of parametrized utility functions, which we call HARA+

$$u(x) = \frac{1}{\rho}\left[(a + hx)^{\rho} - a^{\rho}\right] + bx \tag{17}$$

where $a \geq 0$, $h > 0$, $b \geq 0$, and $0 < \rho < 1$. This expression boils down to the CES for $a = b = 0$, while it is equivalent to a standard HARA utility when $b = 0$. Setting (i) $a = 1$,

$h = 1$, $b \geq 0$, $\rho = 1/2$, and (ii) $a = 0$, $h = 1$, $b = 1$, $\rho = 1/2$ in (17), we get

$$u_1(x) = 2\sqrt{x+1} - 2 \qquad u_2(x) = 2\sqrt{x} + x$$

which imply

$$r_{u_1}(x) = \frac{1}{2(1+1/x)} \qquad r_{u_2}(x) = \frac{1}{2(1+\sqrt{x})}.$$

Clearly, the former increases with x whereas the latter decreases, which means that the market outcome is pro-competitive under u_1 and anti-competitive under u_2. For simplicity, we have chosen to express the equilibrium prices through their inverse $N(\bar{p}_i)$ for $i = 1, 2$ in which E is normalized to 1. Under u_1, we have

$$N(\bar{p}_1) = \frac{2c - \bar{p}_1}{2\bar{p}_1(\bar{p}_1 - c)}$$

which is decreasing in $\bar{p}_1$ over the interval $(c, 2c)$, so that $\bar{p}_1(N)$ also decreases with N. Note that $\bar{p}_1$ is always smaller than $2c$ for a positive mass of firms to be active and tends to c when N tends to infinity.

Under u_2, we obtain

$$N(\bar{p}_2) = \frac{4(\bar{p}_2 - c)^2}{\bar{p}_2(\bar{p}_2 - 2c)^2}$$

which is also defined over $(c, 2c)$. Differentiating this expression shows that $N(\bar{p}_2)$ is increasing on this interval. Consequently, the equilibrium price decreases with N under u_1, whereas it increases under u_2.

This difference in results may be understood as follows. We have seen that the multiplier λ increases with N. Using (2) it is then readily verified that the entry of firms shifts down proportionally the incumbents' inverse demands. Figure 2 shows how the inverse demands change when λ increases from 1 to 2. In the left-handed panel, the marginal revenue curve is sufficiently flat for the equilibrium prices to go down. In contrast, in the right-handed panel, the stronger curvature of the inverse demand implies that the equilibrium price increases with N. Thus, whether the market is pro- or anti-competitive depends on the radial convexity of the inverse demands relative to the one of the CES-demand.

To show it, let $p_1(z)$ and $p_2(z)$ be two inverse demand functions. For any $\alpha \in (0, \pi/2)$, the corresponding radius intersects these two demands curves at some points $z_1(\alpha)$ and $z_2(\alpha)$ which are such that

$$\frac{p_1[z_1(\alpha)]}{z_1(\alpha)} = \frac{p_2[z_2(\alpha)]}{z_2(\alpha)} = \alpha.$$

Multiplying this expression by $p_1'[z_1(\alpha)]/p_2'[z_2(\alpha)]$ and using (9), we obtain

$$r_1[z_1(\alpha)]/r_2[z_2(\alpha)] = e_1[z_1(\alpha)]/e_2[z_2(\alpha)] = p_1'[z_1(\alpha)]/p_2'[z_2(\alpha)].$$

When $r_2(z)$ is constant, $r_1(z)$ is increasing (resp., decreasing) if and only if p_1 is less (resp., more) radially convex than the inverse iso-elastic demand. Thus, *whether the market is pro- or anti-competitive depends on the curvature of varieties' demand relative to the curvature of the CES-demand.*

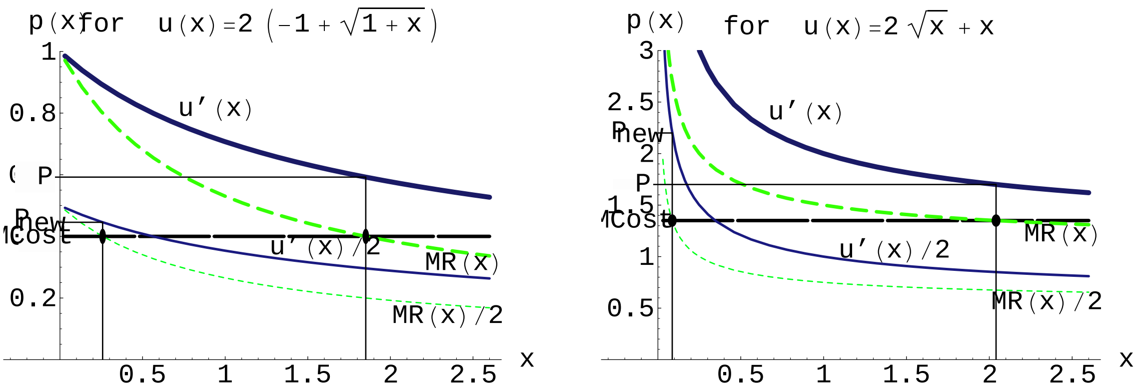

Figure 2: Increase in competition under pro- and anti-competitive inverse demands

2.3 The long-run equilibrium

A symmetric *long-run equilibrium* is defined by a mass of firm $\bar{N}$ and a symmetric equilibrium $\bar{p}$ such that each firm maximizes its profits with respect to its price and earns zero profits:

$$(\bar{p} - c)L\bar{x} = f$$

which shows that the equilibrium outcome depends on the ratio L/f.

In order to write the equilibrium conditions in a more compact way, we set

$$\tilde{L} \equiv \frac{c}{f} L$$

which we call the "cost-adjusted market size." Doing comparative statics in terms of $\bar{L}$ allows one to capture shocks in both population and technology. Combining the zero-profit condition, the budget constraint and the identity

$$\bar{p} = \frac{c}{1 - \bar{M}}$$

we find the equilibrium consumption of each supplied variety:

$$\bar{x} = \frac{1}{\tilde{L}} \left(\frac{1}{\bar{M}} - 1 \right). \tag{18}$$

Furthermore, using the zero-profit condition and the budget constraint, we obtain

$$\frac{E}{\bar{N}} = \frac{F}{L\bar{M}} = \frac{c}{\tilde{L}\bar{M}}.$$

The conditions for a long-run equilibrium may then be rewritten as follows.[8]

Proposition 2 *Every symmetric long-run equilibrium must satisfy the following two conditions:*

$$\bar{M} = r_u \left[\frac{1}{\tilde{L}} \left(\frac{1}{\bar{M}} - 1 \right) \right] \tag{19}$$

$$\bar{N} = \frac{E\,\tilde{L}}{c} \bar{M}. \tag{20}$$

To illustrate, we go back to the above examples of pro-competitive utility u_1 and anti-

[8] Fairly similar expressions are obtained by Krugman (1979) but our formulation will prove to be easier to handle.

competitive utility u_2 and determine the corresponding markups:

$$\bar{M}_1 = \frac{2}{\sqrt{8\tilde{L}+1}+3} \qquad \bar{M}_2 = \frac{1}{2}\left(1 - \frac{1}{\sqrt{\tilde{L}+1}}\right).$$

It is readily verified that the equilibrium prices are, respectively, decreasing and increasing with $L > 1$.

The following remarks are in order. First, since (19) depends only upon the profit margin $\bar{M}$, the equilibrium conditions do not form a system of simultaneous equations. Solving (19) for $\bar{M}$ and plugging the solution into (20) yields the unique equilibrium value $\bar{N}$. Second, whatever the functional form of the utility u, the equilibrium price does not depend on the expenditure level E. This in turn implies that the equilibrium price prevailing in the MC-sector is independent of how the other sectors (if any) behave. This is because the price-elasticity of a firm's demand, hence this firm's profit-maximizing price, depends only upon the degree of love for variety within the MC-sector. As will be seen in the next section, if the economy involves several sectors and consumers preferences are described by a general two-tier utility $U(X, Y, ...)$, the properties of the upper-tier utility U are immaterial for the equilibrium value of $\bar{p}$. Third, once $\bar{M}$ has been found, $\bar{N}$ is a linear and increasing function of the expenditure level E. In other words, the equilibrium mass of firms is proportional to the income share consumers spend on the differentiated good, while the equilibrium price and individual consumption remain the same. Likewise, the equilibrium mass of firms is a linear and increasing function of the mass of workers L. However, both $\bar{p}$ and individual consumption vary with L. Hence, the two parameters describing the size of the MC-sector, i.e. E and L, do not play the same role in determining the market outcome.

In the three sub-sections below, we study the impact of a change in the cost-adjusted market size on the symmetric long-run equilibrium $(\bar{p}, \bar{N})$.

2.3.1 The impact of market size on price

Taking the total differential of (19) with respect to $\tilde{L}$ yield

$$\frac{d\bar{M}}{d\tilde{L}} = -\left(\frac{1}{\tilde{L}^2}\frac{1-\bar{M}}{\bar{M}} + \frac{1}{\tilde{L}} \cdot \frac{1}{\bar{M}^2}\frac{d\bar{M}}{d\tilde{L}}\right) r_u'.$$

Solving for $d\bar{M}/d\tilde{L}$ and multiplying both sides by $\tilde{L}/\bar{M}$, we obtain:

$$\frac{\tilde{L}}{\bar{M}}\frac{d\bar{M}}{d\tilde{L}} = -\frac{\bar{x}}{r_u + \frac{r_u'}{\tilde{L}r_u}} r_u' = -\frac{\bar{x}\,(1-r_u)}{r_u(1-r_u) + r_u'\bar{x}} r_u' = -\frac{\bar{x}\,(1-r_u)}{(2-r_{u'})\,r_u} r_u'$$

where the first and second equalities follow from (18) and (19), while the last one follows from (16).

To determine the impact of parameters L, c and F, we use the following equalities:

$$\frac{\tilde{L}}{\bar{M}}\frac{d\bar{M}}{d\tilde{L}} = \frac{L}{\bar{M}}\frac{d\bar{M}}{dL} = \frac{c}{\bar{M}}\frac{d\bar{M}}{dc} = -\frac{F}{\bar{M}}\frac{d\bar{M}}{dF}. \tag{21}$$

Since $r_u < 1$ and $r_{u'} < 2$, it must be that $d\bar{M}/d\tilde{L}$ and r_u' have opposite signs. Therefore, as in the foregoing, three cases may arise according to the sign of r_u'. For example, when $r_u' > 0$, the equilibrium mark-up decreases with $\tilde{L}$. As a result, the equilibrium price falls when the population size L increases, the level of fixed cost F decreases, or both. These effects are expected because a larger L or a smaller F fosters entry, which here leads to a lower market price. This corresponds to the standard pro-competitive effect generated by a bigger market. In contrast, when $r_u' < 0$, we fall back on the anti-competitive case uncovered in the above section.

The impact of c on $\bar{p}$ is less straightforward. Indeed, when the marginal cost increases, everything else being equal, operating profits are lower so that the economy accommodates fewer firms. However, because $r_u' > 0$, a smaller mass of firms leads to a higher market price. Therefore, the impact of an increase in c generates two opposite effects. In order to determine the global impact, we rewrite the equilibrium markup as follows

$$\bar{p}(c) = \frac{c}{1 - \bar{M}(c)}$$

which leads to

$$\frac{c}{\bar{p}}\frac{d\bar{p}}{dc} = 1 + \frac{\bar{M}}{1 - \bar{M}}\frac{c}{\bar{M}}\frac{d\bar{M}}{dc}.$$

To determine the lower bound of this elasticity, observe that

$$\begin{aligned} \frac{\bar{M}}{1 - \bar{M}}\frac{\tilde{L}}{\bar{M}}\frac{d\bar{M}}{d\tilde{L}} &= -\frac{\bar{M}}{1 - r_u}\frac{(-r_{u'} + 1 + r_u)(1 - r_u) r_u}{(2 - r_{u'}) r_u} \\ &= \frac{r_{u'} - 1 - r_u}{2 - r_{u'}}\bar{M} > \frac{r_{u'} - 2}{2 - r_{u'}}\bar{M} = -\bar{M}. \end{aligned} \tag{22}$$

Using (21), this implies

$$\frac{c}{\bar{p}}\frac{d\bar{p}}{dc} = 1 + \frac{\bar{M}}{1 - \bar{M}}\frac{\tilde{L}}{\bar{M}}\frac{d\bar{M}}{d\tilde{L}} > 1 - \bar{M} > 0. \tag{23}$$

Consequently, the market price always increases with the marginal cost, regardless of the sign of $r_{u'}$. However, the elasticity of $\bar{p}$ with respect to c depends on the sign of $r_{u'}$. Indeed, if $r_{u'} > 0$, we have $d\bar{M}/d\tilde{L} < 0$. In this case, (23) implies that $1 > (c/\bar{p})(d\bar{p}/dc) > 1 - \bar{M}$. Thus, each firm absorbs some fraction of the cost increase. On the other hand, if $r_{u'} < 0$, we have $(c/\bar{p})(d\bar{p}/dc) > 1$, meaning that a higher marginal cost leads to a more proportional increase in market price.

2.3.2 The relationship between market size and industry size

To study how the size of the MC-sector changes with the structural parameters, we differentiate the equilibrium condition (20). Using (22), we obtain

$$\frac{\tilde{L}}{\bar{N}}\frac{d\bar{N}}{d\tilde{L}} = 1 + \frac{\tilde{L}}{\bar{M}}\frac{d\bar{M}}{d\tilde{L}} > \bar{M} > 0.$$

Hence, regardless of the sign of r_u', the equilibrium mass of firms is always an increasing function of the size of the economy. Thus, the RLV does not affect the pro-entry effect generated by a larger market. However, it affects the way this pro-entry effect reacts to market size. Indeed, when $r_u' < 0$ (resp., > 0), the above elasticity exceeds 1, which means that $\bar{N}(L)$ grows at an increasing (resp., decreasing) rate. In other words, *when consumers display an decreasing* (resp., increasing) *RLV, a growing population enjoy a more* (resp., less) *than proportionate mass of varieties.* As it should now be expected, the mass of varieties grows linearly with L if and only if the utility is given by the CES:

$$\bar{N} = (1 - \rho)\frac{L}{F}. \tag{24}$$

How does $\bar{N}$ react to the cost parameters? Since increasing F is tantamount to decreasing $\tilde{L}$, $\bar{N}$ must decreases with F, while $r_u' > 0$ (resp., < 0) implies that the equilibrium mass of varieties decreases at a growing (resp., falling) rate.

Once more, the impact of c is less straightforward. Differentiating (20) with respect to c and using the counterpart of (21), we get

$$\frac{c}{\bar{N}}\frac{d\bar{N}}{dc} = \frac{\tilde{L}}{\bar{M}}\frac{d\bar{M}}{d\tilde{L}} = \frac{c}{\bar{M}}\frac{d\bar{M}}{dc}.$$

Thus, when c rises, the equilibrium mass of firms may go up or down. Specifically, when $r_u' > 0$, the equilibrium price decreases with $\tilde{L}$, which together with (22) imply

$$-(1 - \bar{M}) < \frac{c}{\bar{N}}\frac{d\bar{N}}{dc} < 0$$

whereas we have

$$\frac{c}{\bar{N}}\frac{d\bar{N}}{dc} > 0$$

when $r_u' < 0$. Note that, for $r_u' = 0$, an increase or a decrease in the marginal cost has no impact on the equilibrium mass of firms, as also shown by (24). Once more, we see that *what determines the properties of the market outcome is the variety-loving attitude of consumers.*

The impact of those parameters on the equilibrium consumption and output can be obtained in a similar way by differentiating the corresponding equilibrium conditions (see Appendix A for more details). Two further results are worth mentioning. First, *the consumption of each variety always falls when the size of the economy rises*, the reason being that consumers prefer to spread their consumption over the wider range of varieties that results from the entry of new firms. Second, despite the larger mass of competitors, *a growing population induces each firm to produce more* (resp., less) *if and only if the RLV is increasing* (resp., decreasing). Again, this is because the entry of new firms leads to a lower (resp., higher) market price.

Our results are summarized in the following proposition.

Proposition 3 *In a symmetric long-run equilibrium, we have:*

RLV	$r'_u(x) > 0$	$r'_u(x) = 0$	$r'_u(x) < 0$
$\bar{p} : L/f$	$\downarrow : -\bar{M} < \frac{L/f}{\bar{p}} \frac{d\bar{p}}{d(L/f)} < 0$	$o\ : \frac{L/f}{\bar{p}} \frac{d\bar{p}}{d(L/f)} = 0$	$\uparrow : 0 < \frac{L/f}{\bar{p}} \frac{d\bar{p}}{d(L/f)} \lesseqgtr 1$
$\bar{p} : c$	$\uparrow : 1 - \bar{M} < \frac{c}{\bar{p}} \frac{d\bar{p}}{dc} < 1$	$\uparrow : \frac{c}{\bar{p}} \frac{d\bar{p}}{dc} = 1$	$\uparrow : 1 < \frac{c}{\bar{p}} \frac{d\bar{p}}{dc}$
$\bar{N} : L/f$	$\uparrow : 0 < \frac{L/f}{\bar{N}} \frac{d\bar{N}}{d(L/f)} < 1$	$\uparrow : \frac{L/f}{\bar{N}} \frac{d\bar{N}}{d(L/f)} = 1$	$\uparrow : 1 < \frac{L/f}{\bar{N}} \frac{d\bar{N}}{d(L/f)}$
$\bar{N} : c$	$\downarrow : -(1 - \bar{M}) < \frac{c}{\bar{N}} \frac{d\bar{N}}{dc} < 0$	$o\ : \frac{c}{\bar{N}} \frac{d\bar{N}}{dc} = 0$	$\uparrow : 0 < \frac{c}{\bar{N}} \frac{d\bar{N}}{dc}$
$\bar{x} : L/f$	$\downarrow : -1 < \frac{L/f}{\bar{x}} \frac{d\bar{x}}{d(L/f)} < 0$	$\downarrow : \frac{L/f}{\bar{x}} \frac{d\bar{x}}{d(L/f)} = -1$	$\downarrow : \frac{L/f}{\bar{x}} \frac{d\bar{x}}{d(L/f)} < -1$
$\bar{x} : c$	$\downarrow : -1 < \frac{c}{\bar{x}} \frac{d\bar{x}}{dc} < 0$	$\downarrow : \frac{c}{\bar{x}} \frac{d\bar{x}}{dc} = -1$	$\downarrow : \frac{c}{\bar{x}} \frac{d\bar{x}}{dc} < -1$
$\bar{q} : L/f$	$\uparrow : 0 < \frac{L/f}{\bar{q}} \frac{d\bar{q}}{d(L/f)} < 1$	$o\ : \frac{L/f}{\bar{q}} \frac{d\bar{q}}{d(L/f)} = 0$	$\downarrow : -1 \lesseqgtr \frac{L/f}{\bar{q}} \frac{d\bar{q}}{d(L/f)} < 0$
$\bar{q} : c$	$\downarrow : -1 < \frac{c}{\bar{q}} \frac{d\bar{q}}{dc} < 0$	$\downarrow : \frac{c}{\bar{q}} \frac{d\bar{q}}{dc} = -1$	$\downarrow : \frac{c}{\bar{q}} \frac{d\bar{q}}{dc} < -1$

Regarding the impact of the market size on the equilibrium price, it is worth noting that *the long-run equilibrium inherits the pro- and anti-competitive properties of the short-run equilibrium.* Note that the size of the industry ($\bar{N}$) always grows with L. Indeed, when there are more consumers, the profits of incumbents always increase, thus attracting new firms. As seen above, according to the sign of r'_u, such an entry leads to a lower or higher market price. When $r'_u > 0$, the decrease in market price slows down the entry of new firms. However, since the elasticity of N with respect to L is less than 1, this negative feedback effect cannot outweigh the initial increase in N. Consequently, the market price is established at a level lower than the initial one. On the contrary, when $r_u' < 0$, the feedback effect is positive. This implies that both effects point to the same direction to make the elasticity of N with respect to L bigger then 1, which yields a higher market price. Yet, this price does not become arbitrarily large because the individual consumption of each variety decreases at a rate exceeding 1. Observe that, under the CES, there is no feedback effect because the market price is unaffected by the entry of new firms. Thus, using the CES as a benchmark, when the market size grows, consumers face a smaller range of varieties and lower prices when they display an increasing RLV. In contrast, the range of varieties is wider and prices are higher under a decreasing RLV than under the CES.

Another peculiar feature of the CES is that the equilibrium size of firms ($\bar{q}$) is independent of the market size. Our results show that *firms' size increases in the pro-competitive case.* This is because the industry size grows at a lower pace than the market size while prices go down. On the contrary, firms' size decreases in the anti-competitive case because the mass of firms increases at a more than proportionate rate and charge a higher price. These effects combine to yield a lower output.

As expected, lowering the fixed cost is equivalent to raising market size. Once more, the impact of a lower marginal cost is less straightforward. When c decreases, profits increase and new firms enter. Simultaneously, the incumbents decrease their prices. In the CES-case, the market price is linear in c since the markup is constant. In the pro-competitive case, the market price decreases at a lower pace than in the CES-case because the entry of new firms slows down the price drop. On the contrary, it decreases at a higher pace in the anti-competitive case because the entry of firms pushes the equilibrium price upwards.

Last, under CES preferences, consumers always benefit from a larger market because prices remain constant while more varieties are available. This positive size effect is reinforced in the pro-competitive case because prices go down while the market supplies more varieties. Hence consumers are better-off. In the anti-competitive case, the impact of market size on welfare is not so clear. Indeed, although more varieties are still available, they are priced at a higher level. Therefore, if the equilibrium price increases at a much higher rate than the mass of varieties, one may expect the welfare level to decrease with the size of the market. And indeed, we have found utility functions such that a growing market is detrimental to consumers.

3 The multi-sector economy

Consider first a two-sector economy. The manufacturing sector provides a differentiated good under increasing returns and monopolistic competition, while the agricultural sector provides a homogeneous good under constant returns and perfect competition. Labor is the only production factor; it is perfectly mobile between sectors.

Each individual supplies inelastically one unit of labor and is endowed with preferences defined by

$$\mathcal{U} = U(X, A) = U\left(\int_0^N u\left[x(i)\right] \mathrm{d}i, A\right)$$

where U is increasing and strictly concave, while A denotes the consumption of the homogeneous good. To make sure that both goods X and A are produced at the market outcome, we assume that the marginal utility of each good tends to infinity when its consumption tends to zero.

Because there is perfect competition and constant returns in the agricultural sector, the price of the homogeneous good is equal to the equilibrium wage times a constant that measures the marginal productivity of labor. We then choose the unit of the homogeneous good for this constant to be equal to 1. Last, choosing the homogeneous good as the numéraire implies that the equilibrium wage is equal to 1 since the output of the agricultural sector is always positive. Since profits are zero, the budget constraint is given by

$$\int_0^N p(i)x(i)\mathrm{d}i + A = 1.$$

This optimization problem may be decomposed in two subproblems. First, for any given expenditure $E < 1$, the consumer's program over the differentiated good is

$$\max \int_0^N u\left[x(i)\right] \mathrm{d}i \qquad \text{s.t.} \quad \int_0^N p(i)x(i)\mathrm{d}i = E.$$

As in the previous section, under the assumption of concave profits, we may focus on a symmetric outcome (p, N), so that the optimal value of the foregoing program is

$$v(p, N, E) \equiv Nu\left(\frac{E}{Np}\right).$$

The function v is the indirect utility level derived from consuming the differentiated good at the symmetric outcome. It follows from the properties of u that v is decreasing and convex in p, increasing in N, increasing and concave in E, while $v''_{pN} < 0$ and $v''_{EN} > 0$.

Second, the upper-tier maximization problem may be written as follows:

$$\max_E U\left(v(p, E, N), 1 - E\right)$$

in which $v(p, E, N)$ is the index of the differentiated good consumption. Let $E(p, N)$ be the unique solution to the first-order condition

$$U'_1(\cdot)\nu'_E(\cdot) = U'_2(\cdot). \tag{25}$$

evaluated at the symmetric outcome (p, N). Hence, the first optimization problem becomes

$$\max \int_0^N u\left[x(i)\right] \mathrm{d}i \qquad \text{s.t.} \quad \int_0^N p(i)x(i)\mathrm{d}i = E(p, N).$$

In the special case of the Cobb-Douglas utility, i.e. $\mathcal{U}(X, A) = \alpha \log X + (1 - \alpha) \log A$, the expenditure function $E(p, N)$ may be obtained as follows. Let e_u denote the elasticity of the lower-tier utility u with respect to consumption. After some manipulations the first-order condition (25) yields

$$\frac{1-\alpha}{\alpha} = \frac{1-E}{v}v'_E = \frac{1-E}{E}e(\bar{x})$$

where $\bar{x}$ denotes the long-run equilibrium consumption of every variety. Using (18) shows that $e(\bar{x})$ depends only upon $\bar{p}$, which is itself the unique solution to (19). Therefore, $e(\bar{x})$ is independent of N, so that the equilibrium expenditure on the differentiated good is defined by

$$\bar{E}(\bar{p}) = \frac{e(\bar{x})}{\frac{1-\alpha}{\alpha} + e(\bar{x})} = \frac{e(x(\bar{p}))}{\frac{1-\alpha}{\alpha} + e(x(\bar{p}))}.$$

This expression is tractable enough to be used in comparative static analyses for many specifications of the lower-tier utility u embodied in a Cobb-Douglas upper-tier utility.

When $\mathcal{U}$ is unspecified, it is not possible to derive a closed-form expression for E. However, we are able to derive the main properties of the long-run equilibrium under some reasonable assumptions on U and u. First, since the equilibrium price $\bar{p}$, individual consumption $\bar{x}$ and firm's output $\bar{q}$ are independent of the value of E (see (19)), their properties hold true within our general setting.

Unfortunately, the characterization of the equilibrium mass of varieties is more involved because it depends on E, which now also depends on N and p. In order to determine the properties of $\bar{N}$, we need, therefore, some additional assumptions. Although working with a specific expenditure function $E(p, N)$ is often the relevant empirical strategy, we give in Appendix B sufficient conditions for the utilities U and u to yield an expenditure function $E(p, N)$ that satisfies the following two intuitive properties:

$$0 \leq \frac{p}{E} \cdot \frac{\partial E}{\partial p} < 1 \qquad \frac{N}{E} \cdot \frac{\partial E}{\partial N} < 1. \tag{26}$$

The interpretation of these conditions has some appeal. First, the assumption that X and A are complements in preferences ($U''_{12} \geq 0$) implies that the second and third inequalities hold. Such an assumption on U is fairly natural in a context in which both X and A refer to composite goods. Furthermore, the first inequality also implies some form of complementarity. Indeed, a higher price for the differentiated good leads consumers to spend more on this good, which agrees with the idea that the consumption of both X and A decreases because they are poor substitutes. Last, it is worth stressing that the conditions (26) are easy to check once specific utilities are used in empirical analyses.

The following proposition is then proven in Appendix C.

Proposition 4 *In the two-sector economy, the long-run equilibrium prices, consumption and production vary with market size and cost parameters as in Proposition 3. Furthermore, if (26) holds, the equilibrium mass of varieties increases with the market size.*

It should be clear that, within a similar modeling structure, the argument developed above also applies to the case of several differentiated and homogeneous goods.

4 An application to international trade

The Dixit-Stiglitz model of monopolistic competition has proven to be a very powerful tool for studying the size and direction of trade flows as well as firms' behavior on the international marketplace (Helpman and Krugman, 1985; Feenstra, 2004). We thus find it natural to revisit those issues within the context of our general setting. This will allow us to test the robustness of the main results derived under the CES as well as to uncover new ones.

Consider an economy endowed with two sectors and two countries, H(ome) and F(oreign), of sizes s_H and s_F, respectively, with $s_H \geq L/2$ and $s_H + s_F = L$. They are populated with identical consumers. Manufacturing firms supplying the foreign country incur an iceberg-type trade cost given by $\tau > 1$, which implies that their marginal delivery cost is equal to $\tau c > c$. Our purpose being to investigate how trade costs affect firms' pricing behavior and production in the MC-sector, we follow the literature and isolate this effect by working with a setting in which workers' wage is equalized between countries. This is guaranteed by assuming that the homogeneous good is costlessly traded. Under this assumption, the price of the homogeneous good is equal across countries. When this good is chosen as the numéraire, workers' wage is equal to 1 in both countries. The numbers of firms N^H and N^F in countries H and F are determined by the zero-profit conditions but, as in the preceding section, the equilibrium prices can be determined without knowing the values of N^H and N^F.

The profit function of the firm i located in country k is given by

$$\pi(p_i^{kk}, p_i^{kl}; p^k(\cdot), p^l(\cdot), E) = (p_i^{kk} - c)Lx_i^{kk} + (p_i^{kl} - \tau c)Lx_i^{kl} - F \tag{27}$$

where p_i^{kk} (resp., p_i^{kl}) denotes the domestic price of variety i charged in country $k = H, F$ (resp., the foreign price of this variety in country $l = H, F$ and $l \neq k$), while x_i^{kk} (resp., x_i^{kl}) is the individual consumption of variety i in country k (resp., country l).

4.1 Individual consumption of domestic and foreign varieties

We show below that, for all utility functions U and u, the individual consumption of domestic and foreign varieties obey several general properties that can be brought to the data.

First, applying the profit-maximizing conditions (19) to (27), we obtain

$$p^{kk} = \frac{c}{1 - r_u(x^{kk})} \qquad p^{kl} = \frac{\tau c}{1 - r_u(x^{kl})} \tag{28}$$

where p^{kk} (resp., p^{kl}) denotes the symmetric domestic price in country $k = H, F$ (resp., the symmetric foreign price in country $l = H, F$ and $l \neq k$), while x^{kk} (resp., x^{kl}) is the individual consumption in country k (resp., country l) of a variety produced in k. Using (28), it follows from the first-order condition for utility maximization that

$$\tau \frac{u'(x^{kk})}{u'(x^{lk})} = \tau \frac{p^{kk}}{p^{lk}} = \frac{1 - r_u(x^{lk})}{1 - r_u(x^{kk})}. \tag{29}$$

Setting

$$\phi(x) \equiv u'(x) \left[1 - r_u(x)\right]$$

(29) may be rewritten as follows:

$$\tau \phi(x^{kk}) = \phi(x^{lk}). \tag{30}$$

Note that ϕ is strictly decreasing since

$$\phi'(x) = 2u^{''} + u^{'''} = (2 - r_{u'})u^{''} < 0$$

when $2 > r_{u'}$, that is, when profits are strictly concave. Therefore, since $\tau > 1$, it must be that

$$x^{kk} > x^{lk} \qquad \text{for } k, l = H, F \quad \text{and} \quad k \neq l. \tag{31}$$

In other words, *the individual consumption of a domestic variety always exceeds the individual consumption of a foreign variety.* Regardless of the difference in country sizes, the existence of trade costs suffices to bias consumers' purchases toward locally produced varieties.

Second, substituting the equilibrium prices (28) into the zero-profit conditions, we get

$$\frac{x^{kk} r_u(x^{kk})}{1 - r_u(x^{kk})} s_k + \tau \frac{x^{kl} r_u(x^{kl})}{1 - r_u(x^{kl})} s_l = \frac{f}{c}.$$

Since $s_H \geq s_F$, this implies

$$\varphi(x^{HH}) \geq \varphi(x^{FF}) \tag{32}$$

where

$$\varphi(x) \equiv \frac{x r_u(x)}{1 - r_u(x)} - \tau \frac{y(x) r_u(y(x))}{1 - r_u(y(x))}$$

while $y(x)$ is the unique solution to the consumer equilibrium condition (30) rewritten as follows:

$$\tau \phi(x) = \phi(y).$$

Since $\varphi(x)$ is strictly decreasing in x (see Appendix D.1), it follows from (32) that

$$x^{HH} \leq x^{FF} \tag{33}$$

where the equality holds if and only if $s_H = s_F$. Hence, *the individual consumption of a domestic variety is lower in the bigger country than in the smaller one.* This in turn implies that the price of a domestic variety is always smaller than the price of an imported variety: $p^{kk} \leq p^{lk}$.

Furthermore, using (29) and $x^{HH} \leq x^{FF}$, we obtain

$$\phi(x^{HF}) = \tau \phi(x^{FF}) \leq \tau \phi(x^{HH}) = \phi(x^{FH}).$$

Since $\phi(x)$ is strictly decreasing, it must be that

$$x^{HF} \geq x^{FH} \tag{34}$$

that is, the consumption of a foreign variety is lower in the bigger country than in the smaller one. Again, the equality holds if and only if $s_H = s_F$.

Note that $x^{HH} \leq x^{FF}$ and $x^{FH} \leq x^{HF}$ imply together that *a consumer located in the bigger country always buys less of every variety than a consumer in the smaller one.* This a priori surprising result stems from the fact that the H-consumers spread their consumption over a wider range of domestic varieties, which in turn makes the foreign varieties relatively less attractive.

Last, we show in Appendix D.2 that $x^{HH} > x^{HF}$ always holds. Consequently, using this inequality as well as (31)-(34), it must be that

$$x^{FH} \leq x^{HF} < x^{HH} \leq x^{FF} \tag{35}$$

where all inequalities are strict if and only if $s_H > s_F$.

4.2 International pricing

The above inequalities will allow us to derive a complete characterization of firms' pricing behavior in foreign markets.

We first consider the case of identical countries ($s_H = s_F$). Since both ϕ and φ are strictly decreasing, $x^{HH} = x^{FF}$ and $x^{HF} = x^{FH}$ hold if and only if the two countries are identical. In the pro-competitive case, it then follows from (29) that firms' pricing involves freight-absorption since $p^{HF} = p^{FH} < \tau p^{HH} = \tau p^{FF}$. In other words, there is *reciprocal dumping.* This is because the presence of foreign firms amounts to an increase of local firms competing on each market. As a result, the equilibrium price charged by the domestic firms goes down since individual utilities are pro-competitive, which incites the foreign firms to absorb some fraction of the trade costs.

In contrast, in the anti-competitive case, we have $p^{HF} = p^{FH} > \tau p^{HH} = \tau p^{FF}$, which means that the pass-through exceeds τ. Stated differently, there is *reciprocal reverse dumping.* Indeed, the opening to trade now leads the local firms to set higher prices in H, which permits the foreign firms to charge phantom freights.

When countries are asymmetric ($s_H > s_F$), (28) and (35) imply the following result.

Proposition 5 *In the long-run equilibrium with asymmetric countries and two-way trade, firms' pricing behavior is described as follows:*

$s_H > L/2$	Pro-competitive utility	Anti-competitive utility
	D: in both H and F $x^{FH} < x^{HF} < x^{HH} < x^{FF}$ $p^{FH} < p^{HF} < \tau p^{HH} < \tau p^{FF}$	RD: in both H and F $x^{FH} < x^{HF} < x^{HH} < x^{FF}$ $p^{FH} > p^{HF} > \tau p^{HH} > \tau p^{FF}$

In Figure 3, we describe the evolution of the equilibrium outcome when the asymmetry between countries $s \equiv s_H/s_F$ rises from 1/2, the world population remaining constant. The upper-tier utility is a Cobb-Douglas in which the share of the differentiated good is 0.8, while $u_1(x) = 2\sqrt{x+1}-2$ (the pro-competitive case described in the left panels) and $u_2(x) = 2\sqrt{x}+x$ (the anti-competitive case described in the right panels) are the lower-tier utilities introduced in section 2.2. The values of the parameters are $\tau = 1.3$, $c = 1$, $f = 1$, and $L = 10$ The solid lines describe the individual consumption and price of every domestic variety, whereas the dashed lines represent the individual consumption and price of every imported variety. We start from $s = 1/2$. When a small number of consumers move from F to H, firms in H earn positive profits whereas firms in F make negative profits. As in Krugman (1980), this spurs entry in H and exit in F. Since domestic firms have a bigger effect on the local market than foreign firms, the foregoing results suggest that the now larger (resp., smaller) mass of firms in H (resp., F) negatively (resp., positively) impacts on the consumption of each domestic variety. As a result, the consumption of a variety produced in H (resp., F) decreases (resp., increases), which in turn implies $x^{HH} < x^{FF}$. Likewise, because local inverse demands are shifted downward (resp., upward) in country H (resp., F), it becomes harder for the foreign firms to export to H than to F. More generally, as the gap between the two

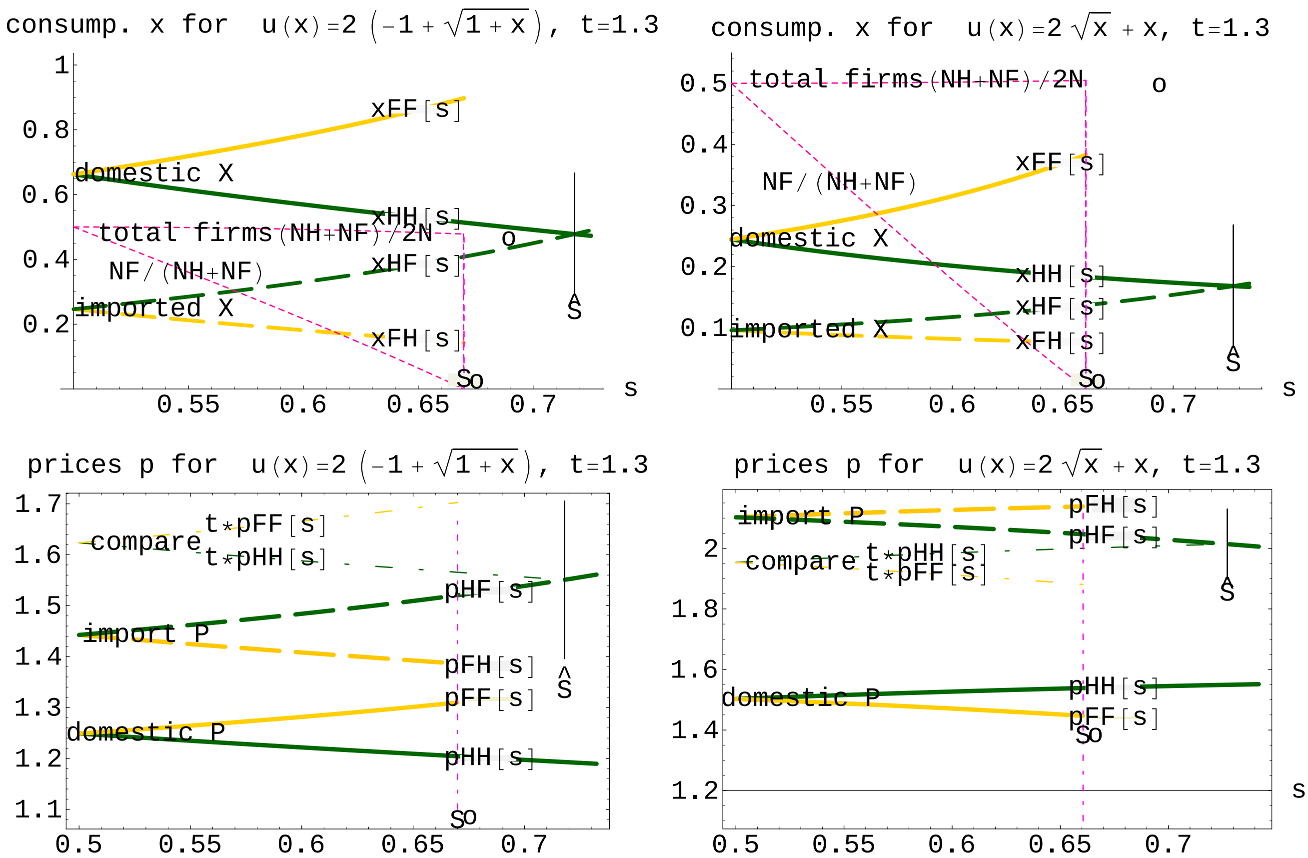

Figure 3: Changes in trade when asymmetry s between the two countries increases.

countries widens, the mass of firms in H rises while the mass of firms in F falls almost lineraly. In country H, consumers face a wider range of domestic varieties, which makes the foreign varieties less attractive. Therefore, in country H the individual consumption of each variety decreases. For exactly the opposite reason, in country F the individual consumption of each variety increases.[9] When countries become sufficiently dissimilar, no firm operates in country F. This is because the local market in F is too small and the market in H too competitive for the operating profits in F to cover their fixed costs evaluated at a wage equal to 1. In other words, for some firms to operate in F, they should pay a wage lower than 1. Since individuals living in F may earn a wage equal to 1 in the agricultural sector, they all choose to work in this sector. Hence, when s exceeds some threshold s_0, *the smaller country no longer produces*

[9]The proof of monotonicity for any utility U and u is long and tedious. It may be obtained from the authors upon request.

the differentiated good, which implies that only intersectoral trade arises.

As shown in the two bottom panels, in the pro-competitive case, the dumping rate practiced by the small country firms ($p^{FH} - \tau p^{FF}$) increases with the asymmetry between countries, whereas the rate of dumping implemented by the large country firms ($p^{HF} - \tau p^{HH}$) decreases. In contrast, when markets are anti-competitive, the reverse dumping policy followed by the F-firms is exacerbated while the reverse dumping policy implemented by the H-firms is weakened. Hence, in both cases, as the asymmetry in countries' size gets larger, domestic and foreign prices diverge more and more.[10]

This discussion does not capture the full richness of firms' pricing behavior on the international marketplace. To sum up, the pricing pattern chosen by firms critically depends on consumers' preferences in a way that vastly differs from what we know from the CES case where the foreign price is equal to the domestic price times τ.

5 Concluding remarks

For monopolistic competition model a method of study is developed and some new effects found.

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Appendix

A. The impact of market size on consumption and production

Consumption. Differentiating (18) and using the counterpart of (21), we get

$$\frac{\tilde{L}}{\bar{x}}\frac{d\bar{x}}{d\tilde{L}} = \frac{L}{\bar{x}}\frac{d\bar{x}}{dL} = \frac{c}{\bar{x}}\frac{d\bar{x}}{dc} = -1 - \frac{1}{1-\bar{M}}\frac{\tilde{L}}{\bar{M}}\frac{d\bar{M}}{d\tilde{L}} < 0. \frac{\tilde{L}}{\bar{x}}\frac{d\bar{x}}{d\tilde{L}} = -1 - \frac{1}{1-\bar{M}}\frac{\tilde{L}}{\bar{M}}\frac{d\bar{M}}{d\tilde{L}} < 0.$$

Hence, the equilibrium consumption of each variety always decreases with $\tilde{L}$, regardless of the sign of $r_{u'}$. Clearly, the same holds when the marginal cost increases, whereas $(F/\bar{x})(d\bar{x}/dF)$ is the same but has the opposite sign. These effects are similar to the one obtained under the CES.

Production. Recall that a firm's production is given by $\bar{q} = L\bar{x} = F\tilde{L}\bar{x}/c$. Thus, the elasticities of $\bar{q}$ with respect to c and F are the same as for $\bar{x}$. As for the impact of L on $\bar{q}$, the sign is a priori undetermined since $\bar{x}$ decreases with L. However, since we have

$$\frac{L}{\bar{q}}\frac{d\bar{q}}{dL} = -\frac{1}{1-\bar{M}}\frac{L}{\bar{M}}\frac{d\bar{M}}{dL}$$

it must be that $d\bar{q}/dL$ and $d\bar{M}/dL$ have opposite signs. Using the counterpart of (21), the impact of F and c is obtained in a similar way through $d\bar{q}/d\tilde{L}$.

B. Properties of the expenditure function

The purpose of this appendix is to prove the following two lemmas. Set

$$D \equiv U''_{11} \cdot \left(\nu'_E\right)^2 - 2U''_{12}\nu'_E + U''_{22} + U'_1\nu''_{EE}.$$

Lemma 1 *If $U''_{21} \geq 0$, then the elasticity of E w.r.t. N is such that*

$$\frac{\partial E}{\partial N} \cdot \frac{N}{E} - 1 = \frac{-U''_{11}\nu'_E\nu + U''_{21}\left(\nu + \nu'_E E\right) - U''_{22}E}{DE} \leq 0.$$

Lemma 2 *If $U''_{21} \geq 0$ and the inequality*

$$\frac{1 - r_u(x)}{e_u(x)} \leq \frac{U''_{21}(X,Y)X}{U'_2(X,Y)} - \frac{U''_{11}(X,Y)X}{U'_1(X,Y)} \tag{B.1}$$

hold at a symmetric outcome, then the elasticity of E w.r.t. p is such that

$$-1 \leq \frac{\partial E}{\partial p} \cdot \frac{p}{E} - 1 = \frac{U'_1\nu'_E + U''_{21}E\nu'_E - EU''_{22}}{DE} \leq 0. \tag{B.2}$$

Remark 1. In the special case of a Cobb-Douglas upper utility, the right-hand side of (B.1) is zero and this condition boils down to

$$1 \leq r_u(x) + e_u(x),$$

which holds true when u is the CES because $r_u(x) = 1 - \rho$ and $e_u(x) = \rho$.

Remark 2. Under $u(0) = 0$, the indirect utility function

$$\nu(p, E, N) = Nu\left(\frac{E}{pN}\right)$$

is homogeneous of degree 0 w.r.t. (p, E) and of degree 1 w.r.t. (E, N). Therefore, ν'_E and ν'_p are homogeneous of degree -1 w.r.t. (p, E) and of degree 0 w.r.t. (E, N). Finally, we have $\nu''_{EE} < 0$.

Before proceeding, recall that the first-order condition for the upper-tier utility maximization (25) is given by

$$U'_1(\nu(p, E, N), 1 - E)\nu'_E(p, E, N) - U'_2(\nu(p, E, N), 1 - E) = 0, \tag{B.3}$$

while the second-order condition is given

$$D < 0.$$

Note that $U(\nu(p,\, E,\, N), 1 - E)$ is concave w.r.t. E as long as U and u are concave.

Proof of Lemma 1. Differentiating (B.3) w.r.t. N and solving for $\partial E/\partial N$, we get

$$\frac{\partial E}{\partial N} = -\frac{U''_{11}\nu'_E\nu'_N + U'_1\nu''_{EN} - U''_{21}\nu'_N}{D} = -\frac{(U''_{11}\nu'_E - U''_{21})\nu'_N + U'_1\nu''_{EN}}{D}.$$

Consequently,

$$\frac{\partial E}{\partial N} \cdot \frac{N}{E} - 1 = -N\frac{(U''_{11}\nu'_E - U''_{21})\nu'_N + U'_1\nu''_{EN}}{DE} - 1 =$$

$$\frac{-U''_{11}\left[\nu'_E N\nu'_N + E\left(\nu'_E\right)^2\right] + U''_{21}\left(N\nu'_N + 2\nu'_E E\right) - U'_1\left(N\nu''_{EN} + E\nu''_{EE}\right) - EU''_{22}}{DE}.$$

Applying the Euler theorem to v and ν', we obtain the following equalities:

$$-U''_{11}\left[\nu'_E N\nu'_N + E\left(\nu'_E\right)^2\right] = -U''_{11}\nu'_E\left(N\nu'_N + E\nu'_E\right) = -U''_{11}\nu'_E\nu$$

$$U_{21}''\left(N\nu_N' + 2E\nu_E'\right) = U_{21}''\left(\nu + E\nu_E'\right)$$

$$-U_1'\left(N\nu_{EN}'' + E\nu_{EE}''\right) = 0.$$

As a result, we have:

$$\frac{\partial E}{\partial N} \cdot \frac{N}{E} - 1 = \frac{-U_{11}''\nu_E'\nu + U_{21}''\left(\nu + E\nu_E'\right) - EU_{22}''}{DE}.$$

Since $U_{21}'' \geq 0$, the numerator of this expression is positive. Since $D < 0$, we have

$$\frac{\partial E}{\partial N} \cdot \frac{N}{E} - 1 \leq 0.$$

■

Proof of Lemma 2.

Step 1. Differentiating (B.3) w.r.t. p and solving for $\partial E/\partial p$, we get

$$\frac{\partial E}{\partial p} = \frac{-U_{11}''\nu_p'\nu_E' - U_1'\nu_{Ep}'' + U_{21}''\nu_p'}{D}. \tag{B.4}$$

which implies

$$\begin{aligned}
\frac{\partial E}{\partial p} \cdot \frac{p}{E} - 1 &= p\frac{-U_{11}''\nu_p'\nu_E' - U_1'\nu_{Ep}'' + U_{21}''\nu_p'}{DE} - 1 \\
&= \frac{-U_{11}''\left[p\nu_p'\nu_E' + E\left(\nu_E'\right)^2\right] - U_1'\left(p\nu_{Ep}'' + E\nu_{EE}''\right) + U_{21}''\left(p\nu_p' + 2E\nu_E'\right) - EU_{22}''}{DE}.
\end{aligned}$$

Applying the Euler theorem to v and ν' yields

$$-U_{11}''\left[p\nu_p'\nu_E' + E\left(\nu_E'\right)^2\right] = -U_{11}''\nu_E'\left(p\nu_p' + E\nu_E'\right) = 0$$

and

$$-U_1'\left(p\nu_{Ep}'' + E\nu_{EE}''\right) = U_1'\nu_E' > 0.$$

Therefore,

$$\frac{\partial E}{\partial p}\cdot\frac{p}{E}-1=\frac{U_1'\nu_E'+U_{21}''E\nu_E'-EU_{22}''}{DE}\leq 0$$

since $U_{21}''\geq 0$. Consequently, the right inequality of (B.2) is proven.

Step 2. To show that $\partial E/\partial p>0$, we rewrite (B.4) as follows:

$$\frac{\partial E}{\partial p}=\frac{\nu_p'}{D}\left(-U_{11}''\nu_E'-U_1'\frac{\nu_{Ep}''}{\nu_p'}+U_{21}''\right). \tag{B.5}$$

By definition of v, we have

$$v_p'=-\frac{Eu'}{p^2}<0 \qquad v_E'=\frac{u'}{p} \qquad \nu_{Ep}''=-\frac{u'}{p^2}-\frac{Eu''}{Np^3}.$$

Since $\nu_p'/D>0$, the sign of $\partial E/\partial p$ is the same as that of the bracketed term of (B.5). Substituting these expressions into (A.2) leads to

$$\begin{aligned}
& -U_{11}''\nu_E'-U_1'\frac{\nu_{Ep}''}{\nu_p'}+U_{21}'' \\
= \;& -U_{11}''\frac{u'}{p}-U_1'\frac{-\frac{u'}{p^2}-\frac{Eu''}{Np^3}}{-\frac{Eu'}{p^2}}+U_{21}'' \\
= \;& -\frac{U_1'}{E}\left[\left(\frac{U_{11}''Nu}{U_1'}-\frac{U_{21}''Nu}{U_2'}\right)\frac{Eu'}{Npu}+1+\frac{Eu''}{Npu'}\right].
\end{aligned}$$

Using $-U_1'/E<0$ and $U_1'\nu_E'(p,E,N)=pU_2'/u'$, it follows from (B.1) that

$$\left(\frac{U_{11}''Nu}{U_1'}-\frac{U_{21}''Nu}{U_2'}\right)\frac{Eu'}{Npu}+1+\frac{Eu''}{Npu'}<0\Longrightarrow\frac{\partial E}{\partial p}>0$$

which implies the left inequality of (B.2).■

C. The impact of market size on the mass of firms

We show that the equilibrium mass of firms decreases with the market size L (see Proposition 4). To this end, we exploit the following implicit relationship between $\bar{M}$ and L:

$$NF=L\bar{M}(L)E(\bar{p}(L),N).$$

Differentiating this expression w.r.t. L, we get

$$\frac{\partial N}{\partial L} \cdot \frac{L}{N} = 1 + \frac{\partial \bar{M}}{\partial L} \frac{L}{\bar{M}} + \frac{\partial E}{\partial p} \frac{\bar{p}}{E} \frac{\partial \bar{p}}{\partial L} \frac{L}{\bar{p}} + \frac{\partial E}{\partial N} \frac{N}{E} \frac{\partial N}{\partial L} \frac{L}{N}$$

which implies

$$\frac{\partial N}{\partial L} \cdot \frac{L}{N} \left(1 - \frac{\partial E}{\partial N} \frac{N}{E}\right) = 1 + \left(1 + \frac{\partial E}{\partial p} \frac{\bar{p}}{E} \frac{\bar{M}}{1 - \bar{M}}\right) \frac{\partial \bar{M}}{\partial L} \frac{L}{\bar{M}}.$$

Using Lemmas 1 and 2 of Appendix B, we obtain

$$\begin{aligned} \frac{\partial N}{\partial L} \cdot \frac{L}{N} \left(1 - \frac{\partial E}{\partial N} \frac{N}{E}\right) &\geq 1 + \left(1 + \frac{\partial E}{\partial p} \frac{\bar{p}}{E} \frac{\bar{M}}{1 - \bar{M}}\right) (\bar{M} - 1) \\ &= \bar{M} \left(1 - \frac{\partial E}{\partial p} \frac{\bar{p}}{E}\right) > 0 \end{aligned}$$

which implies

$$\frac{\partial N}{\partial L} \cdot \frac{L}{N} > 0.$$

■

D. Consumptions in international trade

1. Consumptions of domestic and imported varieties. We have seen in Section 3.2 that x^{HH} and x^{FH} (resp., x^{FF} and , x^{HF}) are related through the following relationship:

$$\tau \phi(x) = \phi(y)$$

where x (resp., y) stands for the equilibrium domestic (resp., foreign) sales while $\phi(x) \equiv u'(x) [1 - r_u(x)]$.

Differentiating the equation

$$\tau u'(x)[1 - r_u(x)] = u'(y) [1 - r_u(y)]$$

w.r.t. x, we obtain

$$\tau[2-r_{u'}(x)]u''(x)=y'(x)[2-r_{u'}(y)]u''(y)$$

which implies

$$y'(x)=\tau\frac{[2-r_{u'}(x)]u''(x)}{u''(y)[2-r_{u'}(y)]}>0 \tag{D.1}$$

since profits are strictly concave.

We are now equiped to show that

$$\varphi(x)=\frac{xr_u(x)}{1-r_u(x)}-\tau\frac{y(x)r_u[y(x)]}{1-r_u[y(x)]}$$

is strictly increasing. Differentiating the first term of $\varphi(x)$ and using (16) yields

$$\left[\frac{xr_u(x)}{1-r_u(x)}\right]'=\frac{[1-r_u(x)]\,r_u(x)+xr_u'(x)}{[1-r_u(x)]^2}=\frac{[2-r_{u'}(x)]r_u(x)}{[1-r_u(x)]^2}.$$

Repeating the same operation with the second term of φ and replacing y' by (D.1), we obtain:

$$\begin{aligned}\varphi'(x) &= \frac{[2-r_{u'}(x)]r_u(x)}{[1-r_u(x)]^2}-\tau y'\frac{[2-r_{u'}(y)]r_u(y)}{[1-r_u(y)]^2}\\ &= \frac{[2-r_{u'}(x)]r_u(x)}{[1-r_u(x)]^2}-\tau^2\frac{u''(x)[2-r_{u'}(x)]r_u(y)}{u''(y)[1-r_u(y)]^2}.\end{aligned}$$

Solving consumer's optimization leads to

$$\tau=\frac{u'(y)\,[1-r_u(y)]}{u'(x)\,[1-r_u(x)]}.$$

Replacing τ by the RHS into $\varphi'(x)$ yields

$$
\begin{aligned}
\varphi'(x) &= [2-r_{u'}(x)]\left\{\frac{r_u(x)}{[1-r_u(x)]^2}-\tau\frac{u''(x)u'(y)r_u(y)}{u'(x)[1-r_u(x)]u''(y)[1-r_u(y)]}\right\} \\
&= \frac{2-r_{u'}(x)}{1-r_u(x)}\left\{\frac{r_u(x)}{1-r_u(x)}-\tau\frac{u''(x)u'(y)r_u(y)}{u'(x)u''(y)[1-r_u(y)]}\right\} \\
&= \frac{2-r_{u'}(x)}{1-r_u(x)}\left\{\frac{xr_u(x)}{x[1-r_u(x)]}-\tau\frac{yr_u(x)}{x[1-r_u(y)]}\right\} \\
&= \frac{r_u(x)[2-r_{u'}(x)]}{x[1-r_u(x)]}\left[\frac{x}{1-r_u(x)}-\tau\frac{y}{1-r_u(y)}\right].
\end{aligned}
$$

The first factor is positive when profits are strictly concave. Replacing τ leads to

$$
\begin{aligned}
\varphi'(x) &\propto \frac{x}{1-r_u(x)}-\frac{u'(y)\,[1-r_u(y)]}{u'(x)\,[1-r_u(x)]}\frac{y}{1-r_u(y)} \\
&= \frac{xu'(x)-yu'(y)}{u'(x)\,[1-r_u(x)]}.
\end{aligned}
$$

Observe that $xu'(x)$ is increasing w.r.t. x when $r_u<1$ because $u'+xu''=u'(1-r_u)$. Therefore, since $x=x^{HH}>y=x^{HF}$ when $\tau>1$, it must be that $xu'(x)-yu'(y)>0$. This in turn implies $\varphi'>0$. ■

2. Consumptions of domestic and exported varieties. We show that, under two-way trade, $x^{HH}>x^{HF}$. The proof involves three steps.

Step 1. Consumers' budgets in H and F are:

$$N^H p^{HH} x^{HH} + N^F p^{FH} x^{FH} = E^H$$

$$N^H p^{HF} x^{HF} + N^F p^{FF} x^{FF} = E^F.$$

Combining these two expressions, we get

$$\frac{N^H}{N^F}p^{HH}x^{HH}+p^{FH}x^{FH}=\frac{E^H}{E^F}\left(\frac{N^H}{N^F}p^{HF}x^{HF}+p^{FF}x^{FF}\right)$$

or

$$\frac{N^H}{N^F} = \frac{\frac{E^H}{E^F} p^{FF} x^{FF} - p^{FH} x^{FH}}{p^{HH} x^{HH} - \frac{E^H}{E^F} p^{HF} x^{HF}}.$$

Since there is two-way trade, the LHS of this expression is positive and finite. Consequently, we have

$$\frac{p^{FH} x^{FH}}{p^{FF} x^{FF}} < \frac{E^H}{E^F} < \frac{p^{HH} x^{HH}}{p^{HF} x^{HF}}. \tag{D.2}$$

We show that this interval is non-empty. (i) $zu'(z)$ is stricly increasing. Indeed, $u'(z) + zu''(z) = u'(z)[1 - r_u(z)]$ is positive because $1 > r_u(z)$ by (12) over the interval of all relevant values of z. (ii) Using the monotonicity of $zu'(z)$ on this interval together with the inequality $x^{HH} > x^{FH}$ (see (31)), we have

$$\frac{p^{HH} x^{HH}}{p^{FH} x^{FH}} = \frac{\frac{x^{HH}}{1-r_u^{HH}}}{\frac{\tau x^{FH}}{1-r_u^{FH}}} = \frac{x^{HH}\left(1 - r_u^{FH}\right)}{\tau x^{FH}\left(1 - r_u^{HH}\right)} = \frac{x^{HH} u'(x^{HH})}{x^{FH} u'(x^{FH})} > 1$$

which means that $p^{HH} x^{HH} > p^{FH} x^{FH}$. Similarly, it can be shown that $p^{FF} x^{FF} > p^{HF} x^{HF}$. Combining these two inequalities leads to

$$\frac{p^{FH} x^{FH}}{p^{FF} x^{FF}} < \frac{p^{HH} x^{HH}}{p^{HF} x^{HF}}.$$

Step 2. In what follows, we need the following inequality:

$$\frac{U_1'(v^H,\, A^H)}{U_2'(v^H,\, A^H)} < \frac{U_1'(v^F,\, A^F)}{U_2'(v^F,\, A^F)} \tag{D.3}$$

where v^i is the indirect utility of the differentiated good in country i and $A^i = 1 - E^i$ the consumption of the homogeneous good in this country. To prove (D.3), we first rewrite (A.1) for each country:

$$U_1'(v^H, 1 - E^H)\left(\nu^H\right)'_E = U_2'(v^H, 1 - E^H) \Leftrightarrow \frac{U_1'(v^H, 1 - E^H)}{U_2'(v^H, 1 - E^H)} = \frac{1}{(\nu^H)'_E}$$

$$U_1'(v^F, 1 - E^F)\left(\nu^F\right)'_E = U_2'(v^F, 1 - E^F) \Leftrightarrow \frac{U_1'(v^F, 1 - E^F)}{U_2'(v^F, 1 - E^F)} = \frac{1}{(\nu^F)'_E} \tag{D.4}$$

where $(\nu)'_E$ is the Lagrange multiplier in the consumers' lower-tier optimization under a given E. Using (1) and (28) yields

$$\left(\nu^i\right)'_E = \frac{u'(x^{ii})}{p^i} = \frac{u'(x^{ii})(1 - r_u(x^{ii}))}{c}.$$

Since $u'(x^{ii})[1 - r_u(x^{ii})]$ is decreasing and $x^{FF} > x^{HH}$ by (33), we have $\left(\nu^H\right)'_E > \left(\nu^F\right)'_E$. This together with (D.4) implies (D.3).

Step 3. The remaining of the proof is by contradiction. If $x^{HH} \leq x^{HF}$, then (D.2) implies

$$\frac{E^H}{E^F} < \frac{p^{HH}x^{HH}}{p^{HF}x^{HF}} \leq \frac{p^{HH}}{p^{HF}} = \frac{1}{\tau} < 1 \Longrightarrow E^H < E^F.$$

Hence, $1 - E^H > 1 - E^F$. Observe that the marginal rate of subsitution between the differentiated and homogeneous goods, $U_1'(X, A)/U_2'(X, A)$, is decreasing in X and increasing in A when (26) holds. Indeed, (26) implies

$$\frac{U_{11}''U_2' - U_1'U_{21}''}{(U_2')^2} < 0 \qquad \frac{U_{12}''U_2' - U_1'U_{22}''}{(U_2')^2} > 0.$$

Therefore,

$$\frac{U_1'(v^H, 1 - E^H)}{U_2'(v^H, 1 - E^H)} > \frac{U_1'(v^H, 1 - E^F)}{U_2'(v^H, 1 - E^F)}.$$

Because $x^{HH} \leq x^{HF}$ and $x^{FH} \leq x^{FF}$ (see (31)), we find

$$\nu^H = N^H u(x^{HH}) + N^F u(x^{FH}) \leq N^H u(x^{HF}) + N^F u(x^{FF}) = \nu^F.$$

Since the marginal rate of substitution is decreasing in v, the above inequality implies

$$\frac{U_1'(v^H, 1 - E^H)}{U_2'(v^H, 1 - E^H)} \geq \frac{U_1'(v^H, 1 - E^F)}{U_2'(v^H, 1 - E^F)} \geq \frac{U_1'(v^F, 1 - E^F)}{U_2'(v^F, 1 - E^F)}$$

which contradicts (D.3). ■.