How sellers' capacities affect equilibrium in finite markets

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1. Motivation on finite markets

Many markets are finite, i.e. consist of a limited number of buyers, sellers and **goods**.

- If all buyers come to the same seller, she can't serve all them because of the lack of available products. This is a **friction**.
- In a frictionless setting, buyers will always purchase from the low-price seller.
- Under the pressure of potential friction, the sellers charging the lowest prices are not necessarily chosen by all buyers and may not make the sales in equilibrium.
- Prices do not unravel downward to marginal cost, as would occur under Bertrand competition.

What is the equilibrium pricing?

Basic setting $n_b \times n_s$ market (Wright et. al 2021)

- $I = \{1, 2, ..., n_b\}$ – the set of buyers.
- Each buyer $i \in I$ has unit demand for some good.
- $J = \{1, 2, ..., n_s\}$ – the set of sellers.
- Each seller $j \in J$ has 1 unit of this indivisible good.
- The value of the object for seller $j \in J$ is equal to $c \geqslant 0$.
- The value of the object for buyer $i \in I$ is equal to $u > c$.

Model: actions and timing

The two-stage game:

1. Sellers simultaneously and independently set prices for their objects. $\mathrm{p} = (p_1, p_2, \ldots, p_{n_s})$, $p_i \in [c, u]$, is a price vector.
2. Buyers, observing prices, simultaneously and independently decide which seller to go and to send requests
3. If a firm $j \in J$ gets exactly one request, she immediately sells her product to this buyer for the announced price.
4. If more than one buyer comes to the firm $j \in J$, it chooses exactly one of them with equal probabilities.

Equilibrium

Authors focus on symmetric SPNE with $p_1 = p_2 = \ldots = p_{n_s}$ and probability $1/n_s$ for every buyer to visit every seller.

The final equilibrium price is

$$p = \frac{\left(1-(1-1/n_s)^{n_b} - \frac{n_b}{n_s}(1-1/n_s)^{n_b-1}\right)u + \frac{n_b}{n_s}(1-1/n_s)^{n_b}c}{1-(1-1/n_s)^{n_b} - \frac{n_b}{n_s}(1-1/n_s)^{n_b-1} + \frac{n_b}{n_s}(1-1/n_s)^{n_b}}$$

Special case: $n_s = n_m = 2$

(Burdett, Shi, and Wright 2001)

- The equilibrium price $p = (u+c)/2$
- The probability of visiting every seller $\gamma = 1/2$
- The expected number of trades $\mu = 3/2$
- The individual trading probabilities $\alpha_b = \alpha_s = 3/4$.

2. Extension: Many products

First attempts:

- (Burdett, Shi, and Wright 2001)
  - analyze 2×2 market with 1 and 2 capacities analytically
  - analyze infinitely large market with 1 and 2 capacities and fixed buyer-seller ratio
- (Lester 2010)
  - does the similar things in terms of the labor market
- (Geromichalos 2012)
  - considers larger capacities, but different mechanism of sales
  - non-uniqueness of prediction, problems with analysis of interaction

Insight from early papers:

greater number of products is profitable for sellers

BSW model modification

NEW: every player has $h \geq 1$ products.

Theorem

In a symmetric subgame perfect equilibrium, all sellers post equal prices

$$p^{ds} = \frac{u + c \times \frac{n_b}{n_s} \times \frac{1}{h} \times \left(1 - \frac{1}{n_s}\right) \times \frac{P(X \leqslant h-1)}{P(Y > h)}}{1 + \frac{n_b}{n_s} \times \frac{1}{h} \times \left(1 - \frac{1}{n_s}\right) \times \frac{P(X \leqslant h-1)}{P(Y > h)}}, \tag{1}$$

where X and Y are random variables equal to the number of successes in the $n_b - 1$ and n_b, respectively, Bernoulli experiments with the probability of success $1/n_s$.

Remark (1)

The multiplier $(1 - \frac{1}{n_s})$ *(marked with blue color) in (1) indicates a term reflecting strategic effect among buyers. It disappears when we extend (Montgomery, 1991) formula for h products per seller using market utility approach.*

Remark (2)

It is easy to see, when $h \geqslant n$, $P(Y > h) = 0$ *and* $P(X \leqslant h - 1) = 1$, *that is why* $p^{ds} = c$. *The intuition behind the result is similar to Bertrand's motives.*

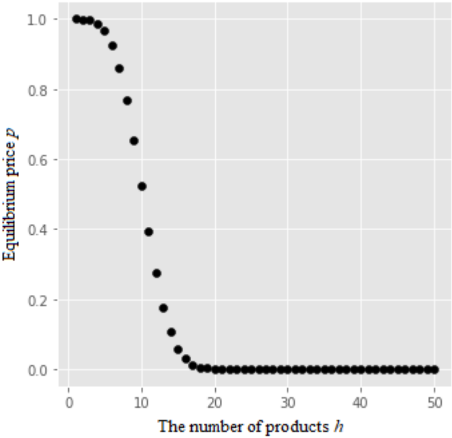

Figure: The equilibrium price in the market with $n_b = 100$, $n_s = 5$, $u = 1$, $c = 0$ as a function of the number of products h

Comparative statics

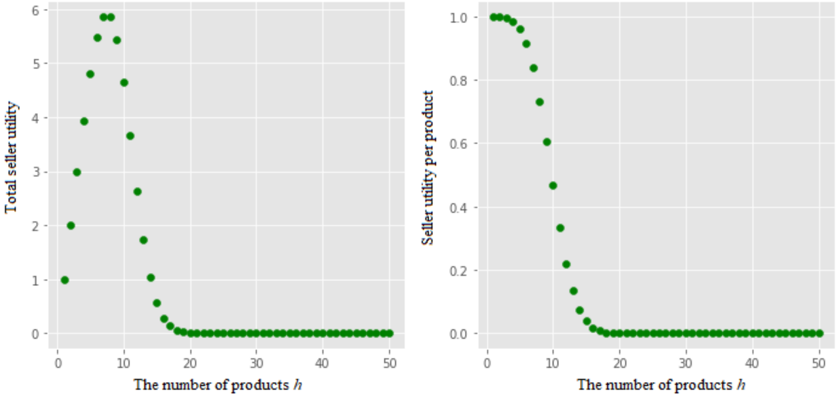

Figure: The equilibrium utility, total and per product, in the market with $n_b = 100$, $n_s = 5$, $u = 1$, $c = 0$ as a function of the number of products h

3. Market concentration

- The effects depend on the number of buyers n_b and the total number of products H.
- $n_b >> H$: with the growth of market concentration, the equilibrium price increases, the utility of every seller increases, and buyer's utility decreases.
- $n_b << H$: with the growth of market concentration, the equilibrium price decreases. The buyer utility and the total number of deals increase. But for sellers a little growth of market concentration is profitable, since it enlarge the demand greater than drops the price.
- $n_b \approx H$: non-monotonic effects
- The efficiency of the market, that is the probability of every product to being sold, increases with the growth of market concentration.

4. Asymmetric capacities

Price vector $p = (p_1, p_2, \dots, p_{n_s})$ and vectors of probabilities to visit every seller $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_{n_s})$ are the solution of the following two systems:

$$\begin{cases} (u - p_1) \times z(\gamma_1, n_b, h_1) & = (u - p_2) \times z(\gamma_2, n_b, h_2), \\ (u - p_2) \times z(\gamma_2, n_b, h_2) & = (u - p_3) \times z(\gamma_3, n_b, h_3), \\ \dots & \\ (u - p_{n_s-1}) \times z(\gamma_{n_s-1}, n_b, h_{n_s-1}) & = (u - p_{n_s}) \times z(\gamma_{n_s}, n_b, h_{n_s}), \end{cases} \tag{2}$$

and

$$\begin{cases} p_1 = \arg\max_{p_1 \in [c,u]} \left[(p_1 - c) \times n_b \times \gamma_1 \times z(\gamma_1, n_b, h_1) \right], \\ \dots \\ p_{n_s} = \arg\max_{n_s \in [c,u]} \left[(p_{n_s} - c) \times n_b \times \gamma_{n_s} \times z(\gamma_{n_s}, n_b, h_{n_s}) \right]. \end{cases} \tag{3}$$

Technical object

The function z reflects the probability of being served at a particular seller

$$z(x, n-1, h) = \sum_{i=0}^{n-1} \underbrace{C_{n-1}^{i} \times x^{i} \times (1-x)^{n-1-i}}_{\text{Prob}(i \text{ buyers out of the rest } n-1 \text{ come to the firm})} \times$$

$$\times \underbrace{\min\left(\frac{h}{i+1}, 1\right)}_{\text{Prob (buyer will be served } | \ i \text{ buyers, except him, came to the firm)}} =$$

$$= P(X \leqslant h-1) + \frac{h}{nx} \times P(Y > h)$$

where X, Y are r.v. equal to the number of successes in the $n-1$ and n Bernoulli experiments with the probability of success x.

Properties

Lemma

The function $z(x,n,h)$ is strictly decreasing in x on $[0,1]$ for $h \leqslant n$.

$$z'(x,n,h) = \begin{cases} 0, \textit{ if } x = 0 \textit{ and } h > 1; \\ \frac{1-n}{2}, \textit{ if } x = 0 \textit{ and } h = 1; \\ -\frac{h}{(n+1)\times x^2} \times P(Y > h), \textit{ if } x \in (0,1]. \end{cases} \quad (4)$$

- Previous papers worked only with restricted $(h = 1)$ version of this function
$$z(x, n-1, 1) = \frac{1-(1-x)^n}{n \times x}.$$
- It is extremely faster to compute $z(x,n,h)$ using incomplete beta functions than using summation. This allows to calculate equilibrium numerically in very large markets in a reasonable time.

Simulations: example

$n_b = 40$ and $n_s = 3$.
$c = 100$, $u = 200$.
Also fix $h_2 = h_3 = 12$.

	$h_1 = 12$	$h_1 = 15$	$h_1 = 20$
Prices	172.4, 172.4, 172.4	161.7, 160.7, 160.7	141.6, 139.5, 139.5
EU_b	23.6	34.96	56.7
E buyers surplus	70.7	104.9	170.2
EU_s	823.7, 823.7, 823.7	857.0, 669.9, 669.9	731.2, 406.0, 406.0
EU_b/h	68.6, 68.6, 68.6	57.1, 55.8, 55.8	36.6, 33.8, 33.8
E seller surplus	2470.9	2196.7	1543.1
E total surplus	2541.6	2301.6	1713.3
# deals / seller	11.4, 11.4, 11.4	13.9, 11.0, 11.0	17.6, 10.3, 10.3
# deals	34.1 (94.8%)	35.9 (92.2%)	38.1 (86.6%)

5. Concluding remarks

Applications

- pricing on labor market platform (in particular, crowdsourcing with homogeneous tasks)
- finite market of medical services (probably, online)

Takeaway

- Additional products at every seller may lead to tougher competition and lower final profits.
- Sellers with larger capacities may set higher prices. However, the additional capacitiçies are not necessary profitable.
- Price is also a signal about availability and a mechanism for discrimination.

Thank you for your attention!

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